Politecnico di Torino
Department of Mechanical and Aerospace Engineering

PROJECT 1

TIRES

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ABSTRACT

This project aims to investigate the main parameters that affect tires behavior. The first step of the project is the evaluation of the stiffness related to longitudinal forces and self-alignment moment. These stiffnesses are used for linear analysis of the vehicle dynamics. Stiffnesses are evaluated by processing the magic formula of the Pacejka's model. The second step is the analysis of the non-linear behavior. The tire characteristics are plot in the full range of slip and slip angle and discussed. The third step is the evaluation of the combined effect of longitudinal and lateral forces.

1. Maximum force coefficient and slope of $F_x - \sigma$ curve

The aim of this task is to determine the slope of the curves $F_x - \sigma$, $F_y - \alpha$ and $M_z - \alpha$. Data have been obtained by the Pacejka's model for different tires.

Since we are group 109, we use tire number 2 (185/60 R14) for our calculations. Expertimental data for the tires are taken from the Excel file "Project 1 - TireCharacteristics - 2022-2023.xlsx".

a. Maximum force coefficient and slope of $F_x - \sigma$ curve

The magic formula of the Pacejka's model provides the longitudinal force F_x as function of slip σ (longitudinal slip)

$$F_x = D\sin\{C\,atan(B(1-E)(\sigma+S_h) + E\,atan[B\,(\sigma+S_h)])\} + S_v$$

with

$$C = b_0$$

$$D = \mu_{xp}F_z = (b_1F_z + b_2)F_z$$

$$BCD_x = (b_3F_z^2 + b_4\,F_z)e^{-b_5\,F_z}$$

$$E = b_6F_z^2 + b_7F_z + b_8$$

$$S_h = b_9F_z + b_{10}$$

$$S_v = 0.$$

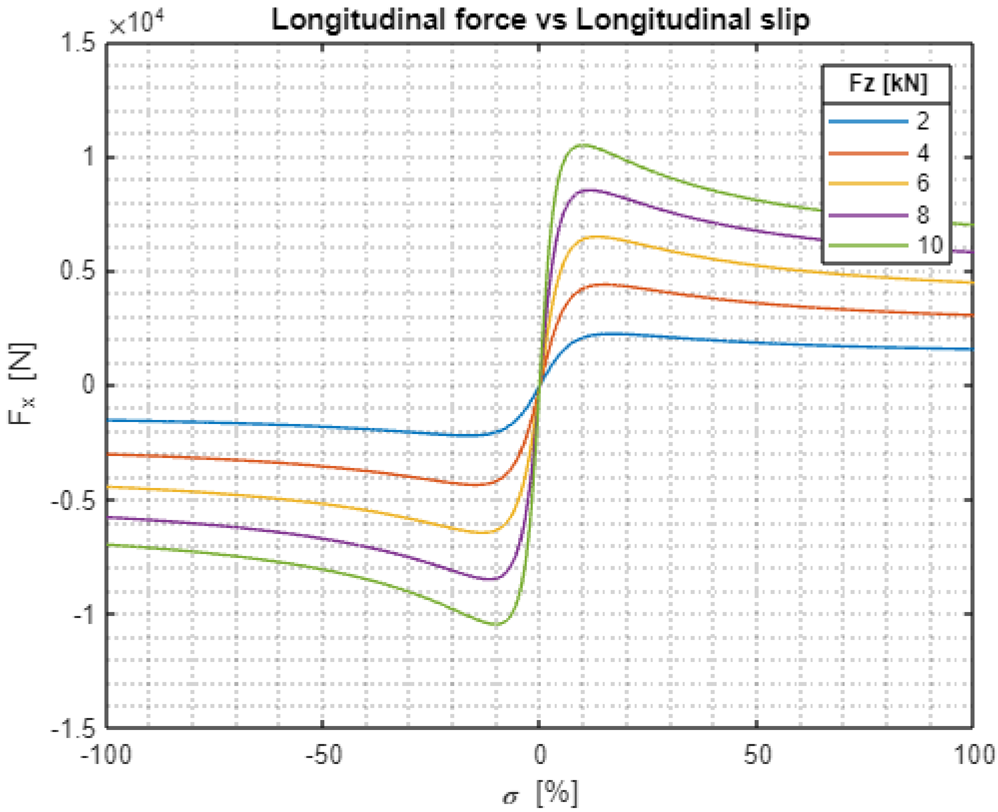
The coefficient *BCDx* is the slope $\left(\frac{\delta F_x}{\delta\sigma}\right)$ of the curve $F_x - \sigma$ at the origin $(\sigma = 0)$.

We start evaluating the longitudinal force as a function of the longitudinal slip; for the calculations we consider the following values of vertical force F_z:

- 2 kN
- 4 kN
- 6 kN
- 8 kN
- 10 kN

```
clear all
close all
clc
% The coefficients for the Pacejka's formula are taken from the Excel file
Coeff = readmatrix('Project 1 - TireCharacteristics - 2022-2023.xlsx', ...
    'Sheet','Tire_2 185 60R14','Range','F3:F13');
b0 = Coeff(1);
b1 = Coeff(2);
b2 = Coeff(3);
b3 = Coeff(4);
b4 = Coeff(5);
b5 = Coeff(6);
b6 = Coeff(7);
```

```
b7 = Coeff(8);
b8 = Coeff(9);
b9 = Coeff(10);
b10 = Coeff(11);

% Then we calculate the vectors of sigma and Fx(sigma) for different values
% of Fz.
sigma = -100:0.1:100;
Fx_Fz_sigma = [];
for Fz=2:2:10
    D = (b1*Fz + b2)*Fz;
    C = b0;
    B = (b3*Fz^2 + b4*Fz)*exp(-b5*Fz) / (D*C);
    E = b6*Fz^2 + b7*Fz + b8;
    Sh = b9*Fz + b10;
    Sv = 0;

    Fx = D.*sin(C.*atan(B.*(1-E).*(sigma + Sh) + E.*atan(B.*(sigma + Sh)))) + Sv;
    Fx_Fz_sigma = [Fx_Fz_sigma; Fx];
end

figure;
plot(sigma, Fx_Fz_sigma); grid minor
title('Longitudinal force vs Longitudinal slip' )
xlabel('\sigma [%]')
ylabel('F_x [N]')
lgd = legend('2','4','6','8','10');
lgd.Title.String = 'Fz [kN]';
```

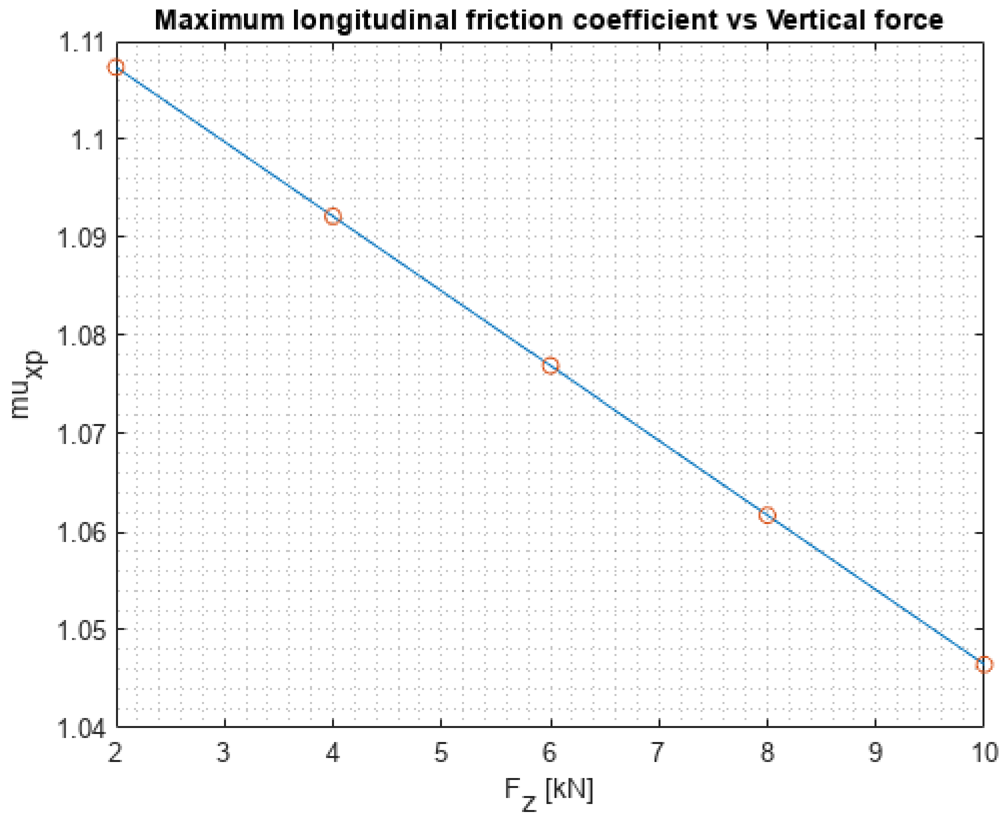


We now calculate the maximum longitudinal force coefficient μ_{xp}.

Using the following equation:

$$D = \mu_{xp} F_z = (b_1 F_z + b_2) F_z$$

```
Fz = 2:2:10;
mu_xp = (b1.*Fz + b2)./1000;
figure; plot(Fz, mu_xp); hold on; plot(Fz, mu_xp, 'o'); grid minor
title('Maximum longitudinal friction coefficient vs Vertical force')
ylabel('mu_{xp}')
xlabel('F_z [kN]')
```

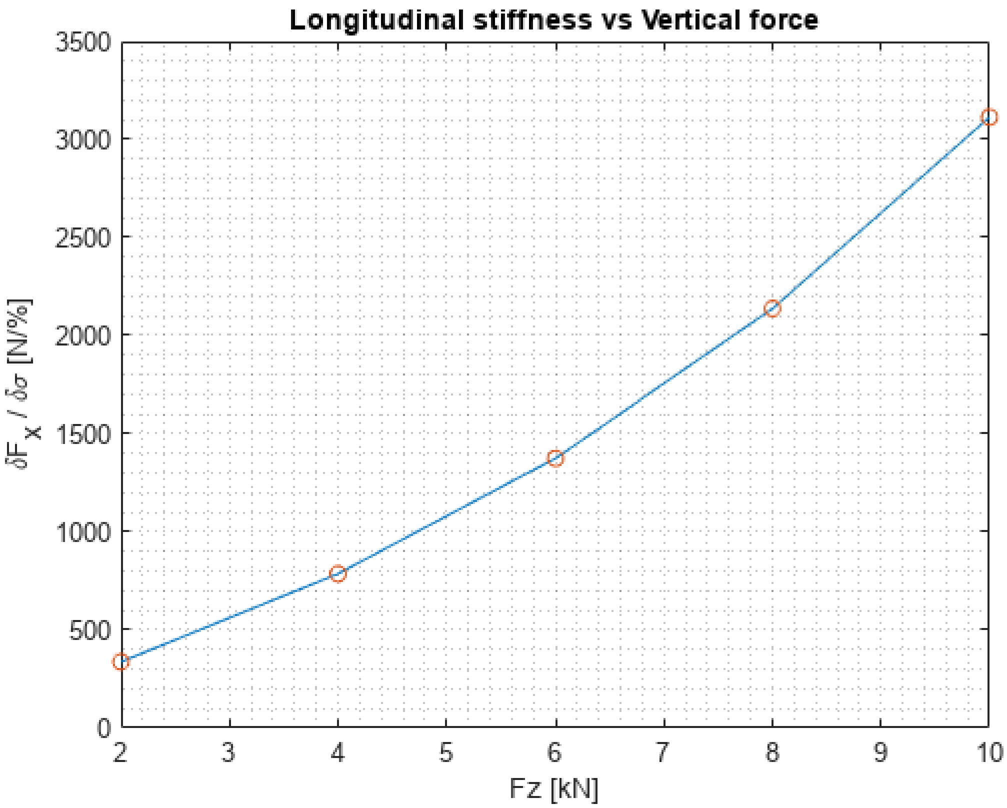


We calculate the slope BCDx $\left(\frac{\delta F_x}{\delta\sigma}\right)$ of the curve $F_x - \sigma$ at the origin $(\sigma = 0)$.

Using the following equation:

$$BCD_x = (b_3 F_z^2 + b_4 F_z)e^{-b_5 F_z}$$

```
Fz = 2:2:10;
BCDx = (b3.*Fz.^2 + b4.*Fz).*exp(-b5.*Fz);
figure;
plot(Fz,BCDx); hold on; plot(Fz,BCDx,'o'); grid minor
title('Longitudinal stiffness vs Vertical force')
xlabel('Fz [kN]')
ylabel('\deltaF_x / \delta\sigma [N/%]')
```



The behaviur of μ_{xp} is coherent with the experimental evidence: an increase of vertical load leads to a decrease of friction coefficient. In terms of force, it means that increasing F_z, the longitudinal force F_x also increases, but with a proportion that is less than linear.

Furthermore, we notice how in $F_x - \sigma$ curves, the higher F_z, the steeper the curve in the origin. This is also shown by the trend of BCD_x as a function of F_z.

b. Maximum force coefficient and slope of $F_y - \alpha$ curve

The Pacejka's magic formula provides lateral force F_y as function of the slip angle α and camber angle γ,

$$F_y = D \sin\{C\ atan(B(1-E)(\alpha + S_h) + E\ atan[B\ (\alpha + S_h)])\} + S_v$$

with

$$C = a_0,$$

$$D = \mu_{yp} F_z = (a_1 F_z + a_2) F_z\ ,$$

$$BCD_y = a_3 \sin\left(2\,\text{atan}\left(\frac{F_z}{a_4}\right)\right)(1 - a_5|\gamma|)\ ,$$

$$E = a_6 F_z + a_7,$$

$$S_h = a_8 \gamma + a_9 F_z + a_{10},$$

$$S_v = a_{11} \gamma F_z + a_{12} F_z + a_{13}.$$

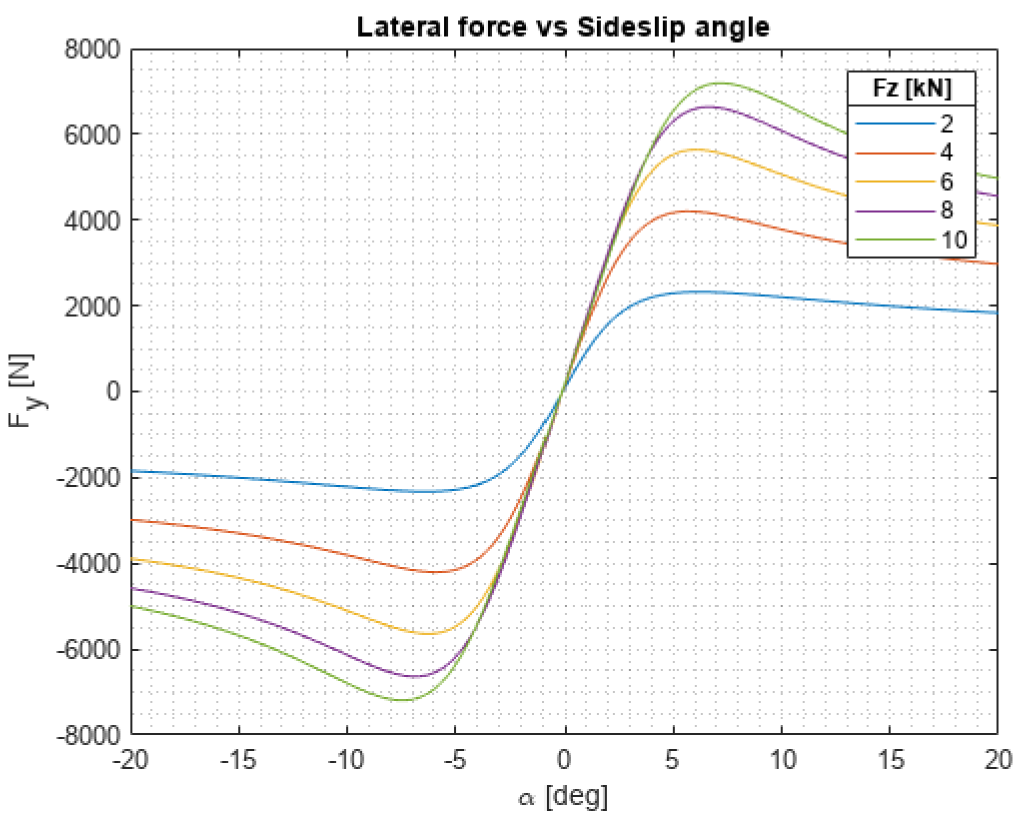
The coefficient a_{11} is given by the relationship

$$a_{11} = a_{111} F_z + a_{112}$$

We proceed evaluating the lateral force as a function of the sideslip angle; for the moment we consider camber angle $\gamma = 0°$, and the same values of F_z.

```
% The coefficients for the Pacejka's formula are taken from the Excel file
Coeff = readmatrix('Project 1 - TireCharacteristics - 2022-2023.xlsx', ...
    'Sheet','Tire_2 185 60R14','Range','D3:D17');
a0 = Coeff(1);
a1 = Coeff(2);
a2 = Coeff(3);
a3 = Coeff(4);
a4 = Coeff(5);
a5 = Coeff(6);
a6 = Coeff(7);
a7 = Coeff(8);
a8 = Coeff(9);
a9 = Coeff(10);
a10 = Coeff(11);
a111 = Coeff(12);
a112 = Coeff(13);
a12 = Coeff(14);
a13 = Coeff(15);

% Then we calculate the vectors of alpha and Fy(alpha) for different values
% of Fz.
alpha = -20:0.1:20;
```

```
gamma = 0;
Fy_Fz_alpha = [];
for Fz=2:2:10
    C = a0;
    D = (a1*Fz + a2)*Fz;
    B = (a3*sin(2*atan(Fz/a4))*(1-a5*abs(gamma)))/(C*D);
    E = a6*Fz + a7;
    Sh = a8*gamma + a9*Fz + a10;
    Sv = (a111*Fz + a112)*gamma*Fz + a12*Fz + a13;

    Fy = D.*sin(C.*atan(B.*(1-E).*(alpha + Sh) + E.*atan(B.*(alpha + Sh)))) + Sv;
    Fy_Fz_alpha = [Fy_Fz_alpha; Fy];
end


figure;
plot(alpha, Fy_Fz_alpha); grid minor
title('Lateral force vs Sideslip angle')
xlabel('\alpha [deg]')
ylabel('F_y [N]')
lgd = legend('2','4','6','8','10');
lgd.Title.String = 'Fz [kN]';
```

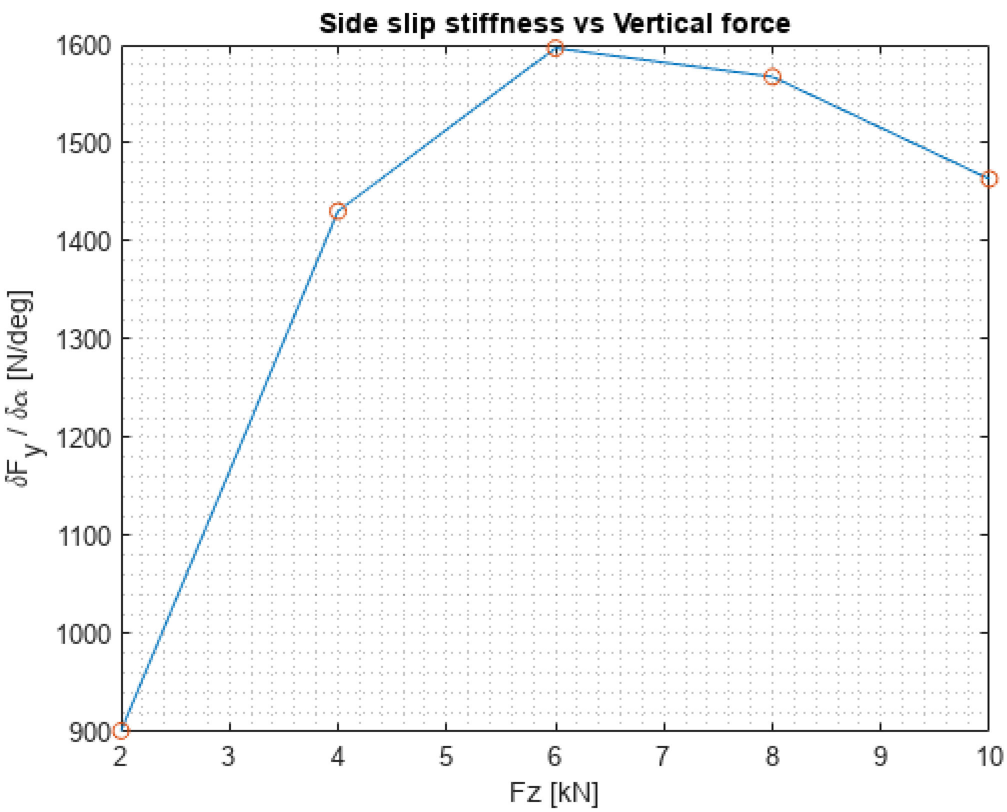


We compute the side slip stiffness BCDy with $\gamma = 0^\circ$.

Using the following equation:

$$BCD_y = a_3 \sin\left(2\,\mathrm{atan}\left(\frac{F_z}{a_4}\right)\right)(1 - a_5|\gamma|)$$

```
Fz = 2:2:10;
BCDy = a3*sin(2*atan(Fz./a4))*(1-a5*abs(gamma));
figure;
plot(Fz,BCDy); hold on; plot(Fz, BCDy, 'o'); grid minor
title('Side slip stiffness vs Vertical force')
xlabel('Fz [kN]');
ylabel('\deltaF_y / \delta\alpha [N/deg]');
```

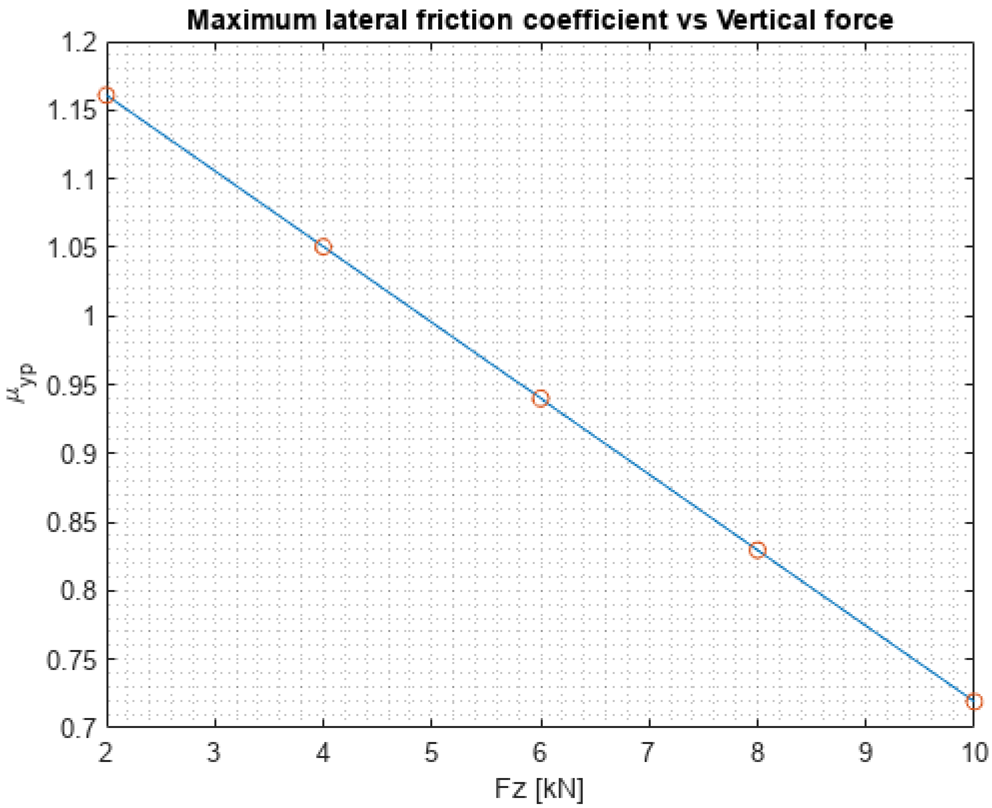


Then we calculate the maximum lateral force coefficient μ_{yp}.

Using the following equation:

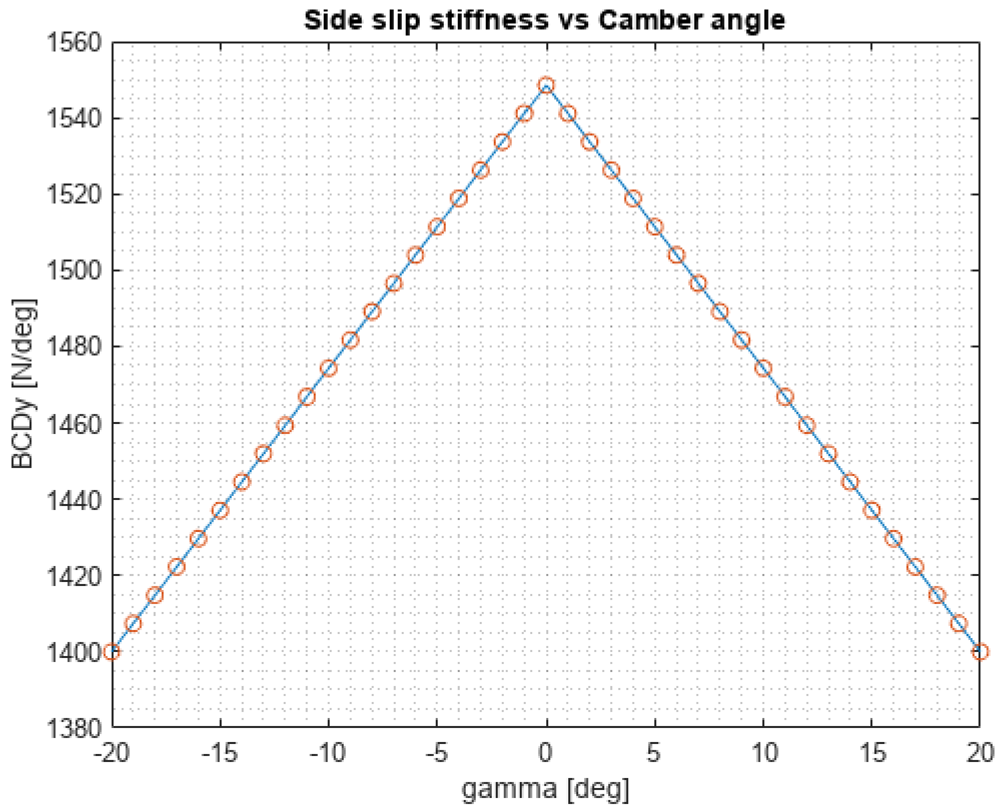
$$D = \mu_{yp} F_z = (a_1 F_z + a_2) F_z$$

```
Fz = 2:2:10;
mu_yp = (a1.*Fz + a2)./1000;
figure; plot(Fz, mu_yp); hold on; plot(Fz, mu_yp, 'o'); grid minor
title('Maximum lateral friction coefficient vs Vertical force')
ylabel('\mu_{yp}')
xlabel('Fz [kN]')
```



For the vertical force $F_z = 5\text{kN}$, we evaluate the effect of the camber angle γ on the side slip stiffness BCDy varying it between $\pm 20°$.

```
Fz = 5;
gamma = -20:1:20;
BCDygamma = a3.*sin(2.*atan(Fz/a4)).*(1-a5.*abs(gamma));
figure;
plot(gamma,BCDygamma); hold on; plot(gamma, BCDygamma, 'o'); grid minor
title('Side slip stiffness vs Camber angle')
xlabel('gamma [deg]');
ylabel('BCDy [N/deg]');
```



The following considerations can be made:

- The stiffness BCD_y shows an initial growth with F_z, and then a saturation region with a slight decrease; this is confirmed by experimental evidence;
- As for longitudinal coefficient, also in this case μ_y decreases by increasing F_z;
- A camber angle has a small influence on the lateral stiffness in the origin, especially for small values γ; the main effect of a camber angle on the $F_y - \alpha$ curves is to shift them vertically with, as shown, a slight reduction of slope in the origin.

c. Self-alignment torque stiffness

The Magic Formula gives the self-alignment torque as

$$M_z = D\sin\{C\, atan(B(1-E)(\alpha + S_h) + E\, atan[B\,(\alpha + S_h)])\} + S_v\,,$$

with

$$C = c_0,$$

$$D = c_1 F_z^2 + c_2 F_z,$$

$$E = (c_7 F_z^2 + c_8 F_z + c_9)(1 - c_{10}\gamma),$$

$$BCD_z = -(c_3 F_z^2 + c_4 F_z)(1 - c_6|\gamma|)e^{-c_5 F_z},$$

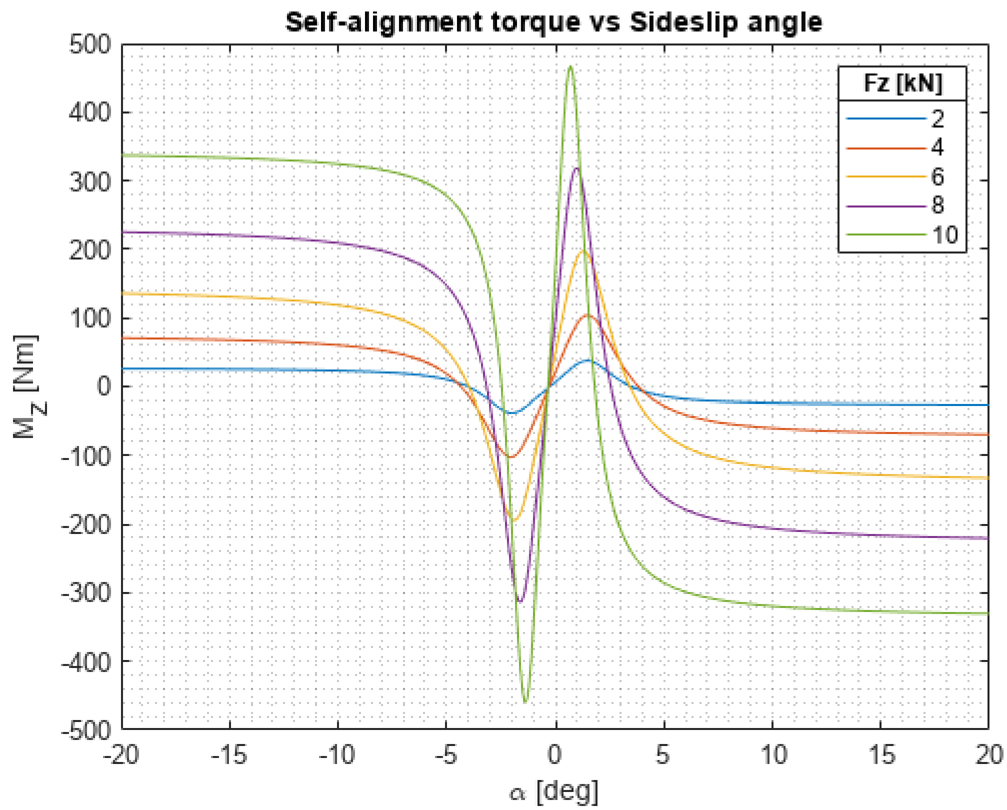
$$S_h = c_{11}\gamma + c_{12} F_z + c_{13},$$

$$S_v = (c_{14} F_z^2 + c_{15} F_z)\gamma + c_{16} F_z + c_{17}.$$

Finally we evaluate the self-alignment torque as a function of the sideslip angle; for the moment we consider camber angle $\gamma = 0°$, and the same values of F_z.

```
% The coefficients for the Pacejka's formula are taken from the Excel file
Coeff = readmatrix('Project 1 - TireCharacteristics - 2022-2023.xlsx', ...
    'Sheet','Tire_2 185 60R14','Range','H3:H20');
c0 = Coeff(1);
c1 = Coeff(2);
c2 = Coeff(3);
c3 = Coeff(4);
c4 = Coeff(5);
c5 = Coeff(6);
c6 = Coeff(7);
c7 = Coeff(8);
c8 = Coeff(9);
c9 = Coeff(10);
c10 = Coeff(11);
c11 = Coeff(12);
c12 = Coeff(13);
c13 = Coeff(14);
c14 = Coeff(15);
c15 = Coeff(16);
c16 = Coeff(17);
c17 = Coeff(18);

% Then we calculate the vectors of alpha and Mz(alpha) for different values
% of Fz.
```

```
alpha = -20:0.1:20;
Mz_Fz_alpha = [];
gamma = 0;
for Fz=2:2:10
    C = c0;
    D = c1*Fz^2 + c2*Fz;
    E = (c7*Fz^2 + c8*Fz + c9)*(1-c10*gamma);
    B = -((c3*Fz^2 + c4*Fz)*(1 - c6*abs(gamma))*exp(-c5*Fz))/(C*D);
    Sh = c11*gamma + c12*Fz + c13;
    Sv = (c14*Fz^2 + c15*Fz)*gamma + c16*Fz + c17;

    Mz = D.*sin(C.*atan(B.*(1-E).*(alpha + Sh) + E.*atan(B.*(alpha+ Sh)))) + Sv;
    Mz_Fz_alpha = [Mz_Fz_alpha; Mz];
end

figure
plot(alpha, Mz_Fz_alpha); grid minor
title('Self-alignment torque vs Sideslip angle')
xlabel('\alpha [deg]')
ylabel('M_z [Nm]')
lgd = legend('2','4','6','8','10');
lgd.Title.String = 'Fz [kN]';
```



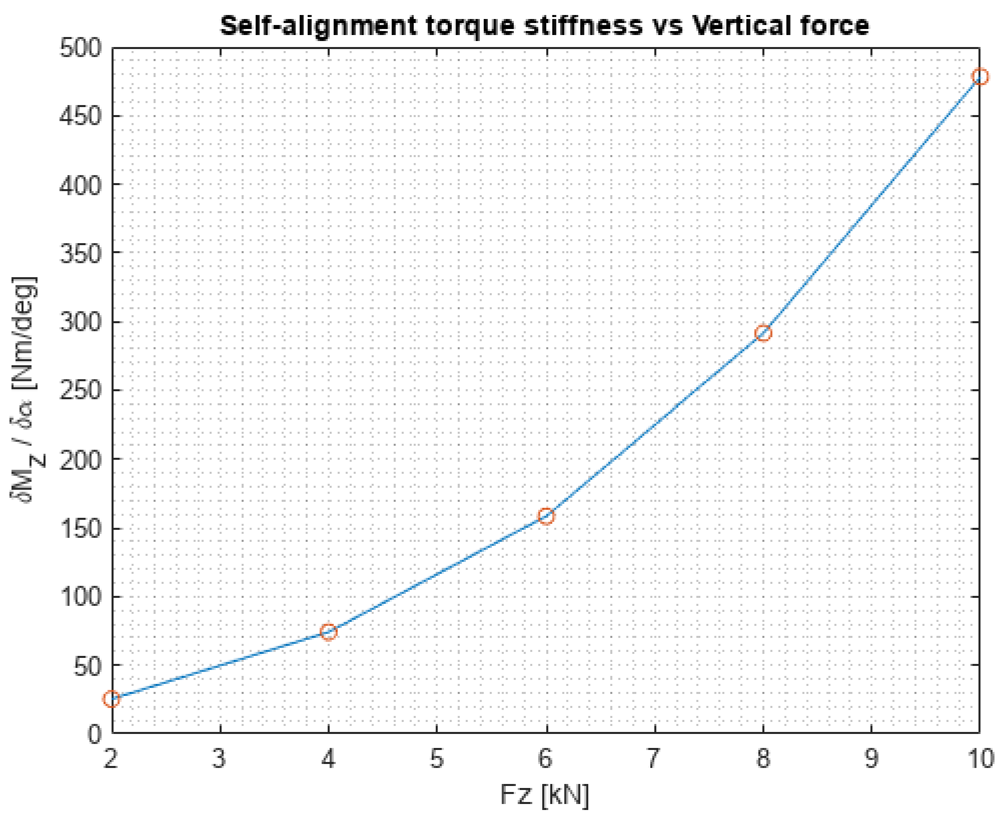
We compute the stiffness BCDz of the self-alignment torque.

Using the following equation:

$$BCD_z = -(c_3 F_z^2 + c_4 F_z)(1 - c_6|\gamma|)e^{-c_5 F_z}$$

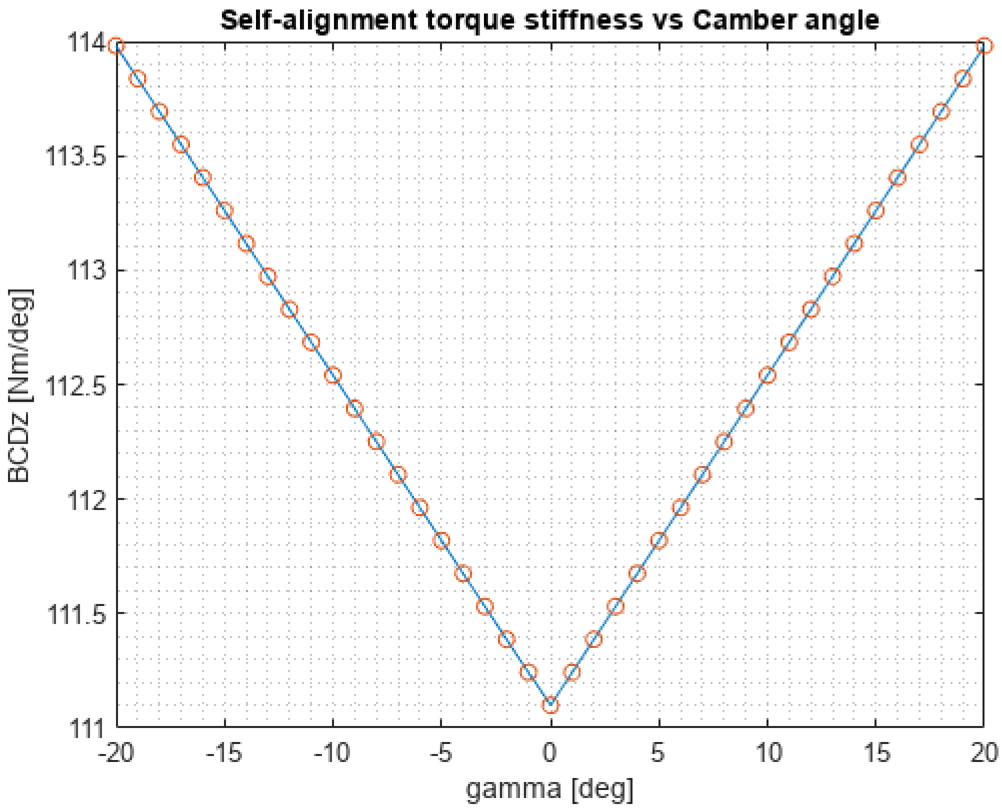
The coefficient is the slope $\left(\frac{\delta M_z}{\delta \alpha}\right)$ of the curve $M_z - \alpha$ at the origin $(\alpha = 0°)$ and also depends on γ.

```
Fz = 2:2:10;
BCDz = -(c3*Fz.^2 + c4.*Fz)*(1 - c6*abs(gamma)).*exp(-c5.*Fz);
figure;
plot(Fz,BCDz); hold on; plot(Fz, BCDz, 'o'); grid minor
title('Self-alignment torque stiffness vs Vertical force')
xlabel('Fz [kN]');
ylabel('\deltaM_z / \delta\alpha [Nm/deg]');
```



For $F_z = 5\text{kN}$, we now consider the effect of the camber angle γ on the self aligning-torque stiffness BCDz varying it between $\pm 20°$.

```
Fz = 5;
gamma = -20:1:20;
BCDzgamma = -(c3*Fz^2 + c4*Fz)*(1 - c6*abs(gamma))*exp(-c5*Fz);
figure;
plot(gamma,BCDzgamma); hold on; plot(gamma, BCDzgamma, 'o'); grid minor
title('Self-alignment torque stiffness vs Camber angle')
xlabel('gamma [deg]');
ylabel('BCDz [Nm/deg]');
```



Also in this case we can make some considerations:

- If no caster trail is foreseen, the self-alignment torque shows a change of sign due to the change of shear stress distibution in the contact patch varying α; this behaviour could be undesired;
- The slope of the $M_z - \alpha$ curves in the origin increases by increasing F_z; this is coherent with the behaviour shown in the $\mathrm{BCD}_z - F_z$ diagram;
- A change in the camber angle shows a very small influence on the self-alignment torque stiffness, with a range of variation of about $3\ \dfrac{\mathrm{Nm}}{\mathrm{deg}}$ for $\gamma = 20°$; in this case, though, the slope tends to increase, maybe due to the fact that a camber angle tends to further deform the contact patch, increasing the asymmetry in the distribution of the shear stress, and thus the lever arm.

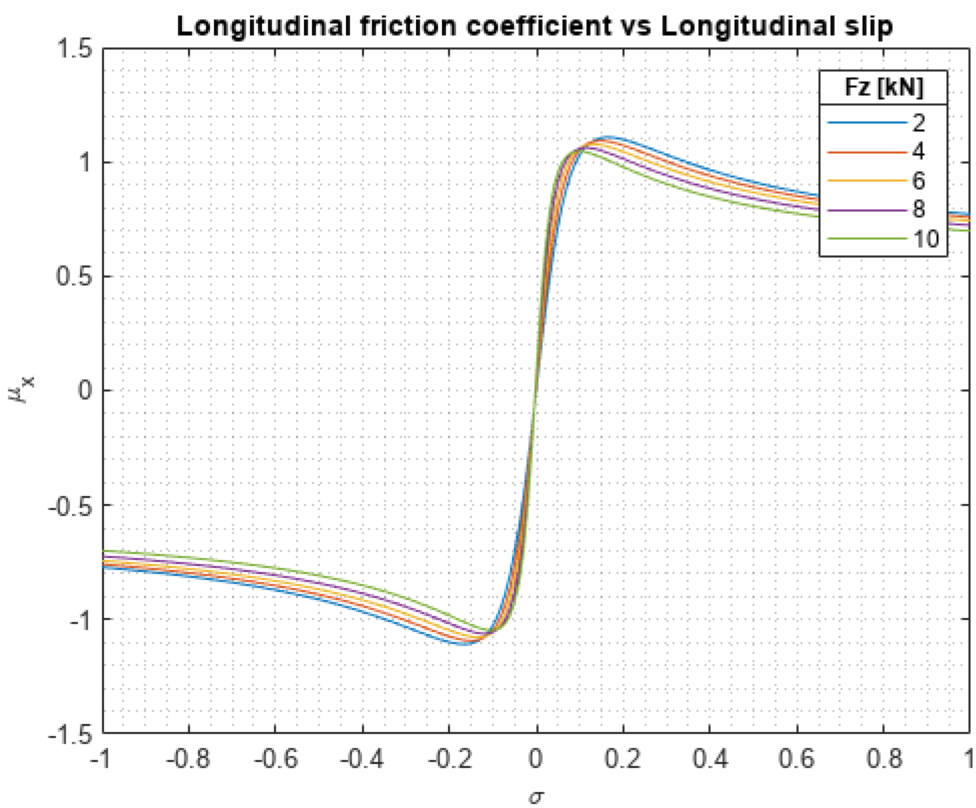
2. TIRE NONLINEAR MODEL

The Pacejka's model provides an approximation for the tire behaviour in the conditions shown; we now want to retrieve the same characteristics basing on the experimental data provided for our tire model.

We start retrieving the longitudinal friction coefficient μ_x dividing the values of longitudinal force F_x by the corresponding vertical force F_z.

```
% The experimental data are taken from the Excel file
sigma_nonlin = readmatrix('Project 1 - TireCharacteristics - 2022-2023.xlsx', ...
    'Sheet', 'Tire_2 185 60R14', 'Range', 'J3:J203');
Fx_matrix = readmatrix('Project 1 - TireCharacteristics - 2022-2023.xlsx', ...
```

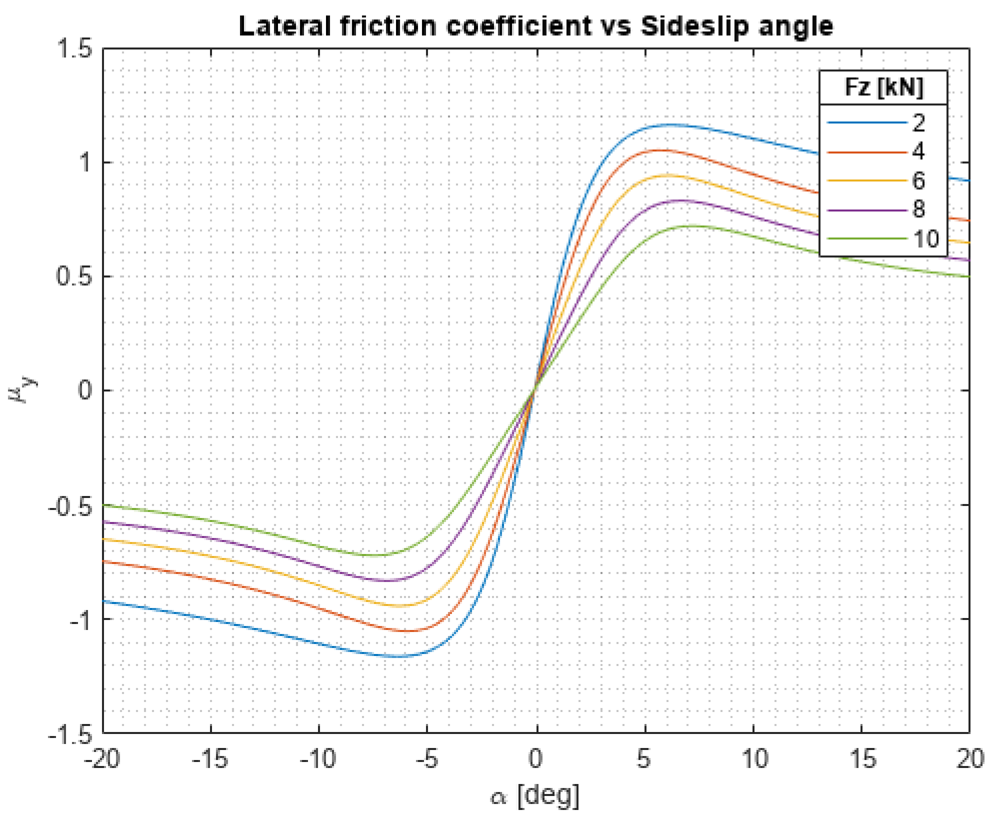
```
    'Sheet', 'Tire_2 185 60R14', 'Range', 'K3:O203');

% We obtain all the values of mux dividing each Fx by the corresponding Fz
index = 0;
mu_x = [];

for Fz = 2 : 2 : 10
    index = index+1;
    mu_x = [mu_x, Fx_matrix(:,index)/ (Fz*10^3)];
end

figure;
plot(sigma_nonlin, mu_x); grid minor
title('Longitudinal friction coefficient vs Longitudinal slip')
xlabel('\sigma');
ylabel('\mu_x');
lgd = legend('2', '4', '6', '8', '10');
lgd.Title.String = 'Fz [kN]';
```



Then we calculate the lateral friction coefficient μ_y dividing the values of lateral force F_y by the corresponding vertical force F_z.

```
% The experimental data are taken from the Excel file
alpha_nonlin = readmatrix('Project 1 - TireCharacteristics - 2022-2023.xlsx', ...
    'Sheet', 'Tire_2 185 60R14', 'Range', 'Q3:Q203');
Fy_matrix = readmatrix('Project 1 - TireCharacteristics - 2022-2023.xlsx', ...
```

```
    'Sheet', 'Tire_2 185 60R14', 'Range', 'R3:V203');

% We obtain all the values of muy dividing each Fy by the corresponding Fz
index = 0;
mu_y = [];

for Fz = 2 : 2 : 10
    index = index+1;
    mu_y = [mu_y, Fy_matrix(:,index)/ (Fz*10^3)];
end

figure;
plot(alpha_nonlin, mu_y); grid minor
title('Lateral friction coefficient vs Sideslip angle')
xlabel('\alpha [deg]')
ylabel('\mu_y')
lgd = legend('2', '4', '6', '8', '10');
lgd.Title.String = 'Fz [kN]';
```



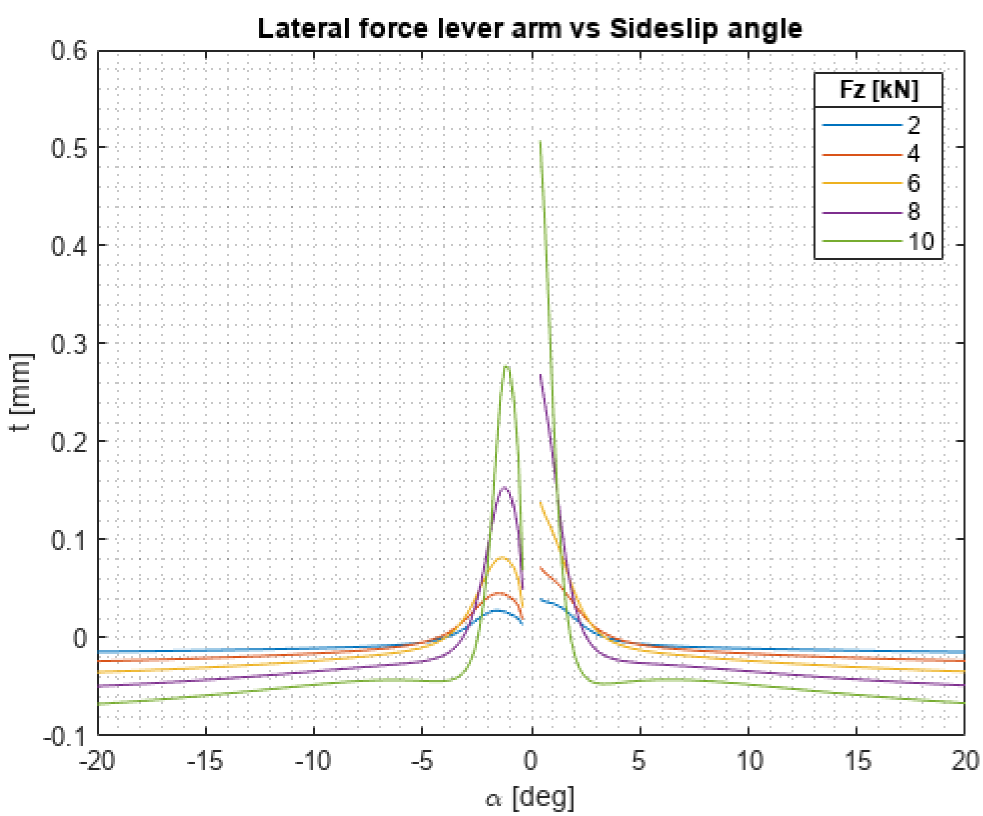
To evaluate the lever arm t of the lateral force as a function of the side slip angle α, for different vertical forces F_z, we divide each value of self-alignment torque M_z by the corresponding lateral force F_y.

```
% The experimental data are taken from the Excel file
Mz_matrix = readmatrix('Project 1 - TireCharacteristics - 2022-2023.xlsx', ...
    'Sheet', 'Tire_2 185 60R14', 'Range', 'Y3:AC203');
```

```
% We obtain all the values of t dividing each Mz by the corresponding Fy
t = [];
index = 0;

for Fz = 2 : 2 : 10
    index = index + 1;
    t = [t, Mz_matrix(:,index) ./ Fy_matrix(:,index)];
end
t(100:102 , :) = NaN; % This command is useful to neglect wrong
                      % values of the lever arm near to the origin
```

SPOILER: Due to the fact that next to the origin the program had to compute fractions between very small numbers, this region of the diagram showed very high values of t, theorically growing to ∞. This has clearly no physical meaning, so we decided not to consider a few values that showed this behaviour.

```
figure;
plot(alpha_nonlin, t); grid minor
title('Lateral force lever arm vs Sideslip angle')
xlabel('\alpha [deg]')
ylabel('t [m]')
lgd = legend('2', '4', '6', '8', '10');
lgd.Title.String = 'Fz [kN]';
```

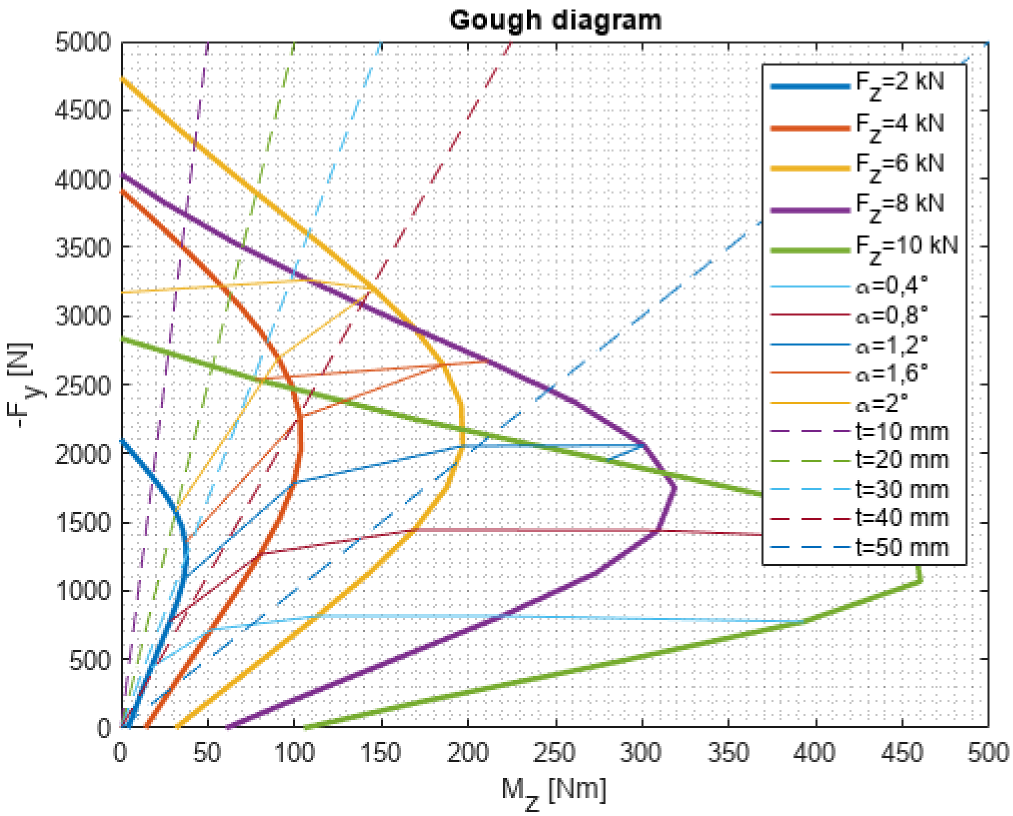


The Gough diagram shows the relation between the self-alignment torque M_z and the lateral force F_y; it is though obtained plotting the corresponding experimental values of these two quantities, for each level of vertical force.

In addition, two other sets of curves are added:

- Constant sideslip angle (thin lines);
- Constant lever arm (dashed lines);

```
figure
plot(Mz_matrix, Fy_matrix, 'LineWidth',2); grid minor % Gough
hold on


% Constant alpha
for index = 103 : 2 : 111
    Fy_alpha = Fy_matrix(index, :);
    Mz_alpha = Mz_matrix(index, :);
    hold on
    plot(Mz_alpha,Fy_alpha);
end

% Constant lever arm
t = [10, 20, 30, 45, 100]*10^-3;
Mz_t = 0:500;
for index = 1: length(t)
    Fy_t= Mz_t/t(index);
    hold on
    plot(Mz_t,Fy_t,'--');

end

lgd = legend('F_z=2 kN', 'F_z=4 kN', 'F_z=6 kN', 'F_z=8 kN', 'F_z=10 kN', ...
    '\alpha=0,4°', '\alpha=0,8°', '\alpha=1,2°', '\alpha=1,6°', '\alpha=2°', ...
    't=10 mm','t=20 mm','t=30 mm','t=40 mm','t=50 mm');
title('Gough diagram')
xlabel('M_z [Nm]');
xlim([0,inf]);
ylabel('-F_y [N]')
ylim([0, 5000]);
```

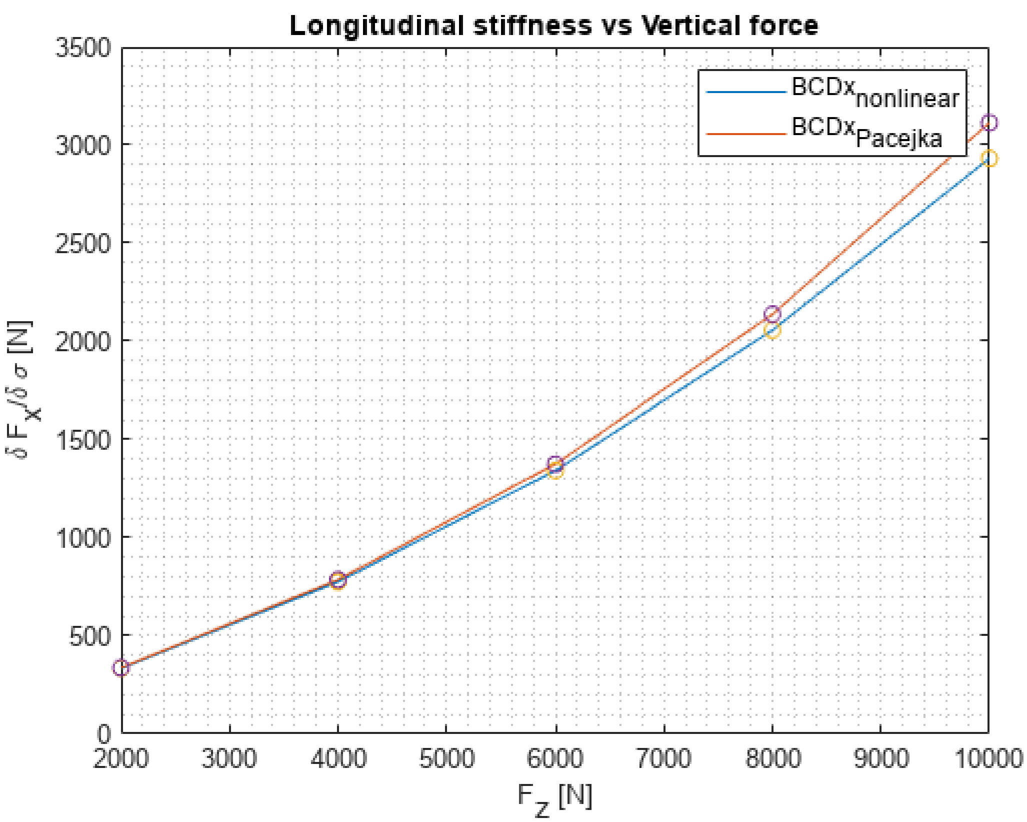


We now compare the slope of the curves with stiffnesses BCD calculated in the first part of the project. Such values represent the slope of the curves in the origin. To obtain this parameter with our data, we just compute the derivative analitically, as the incremental ratio of two consecutive points, choosing the index corresponding to the origin.

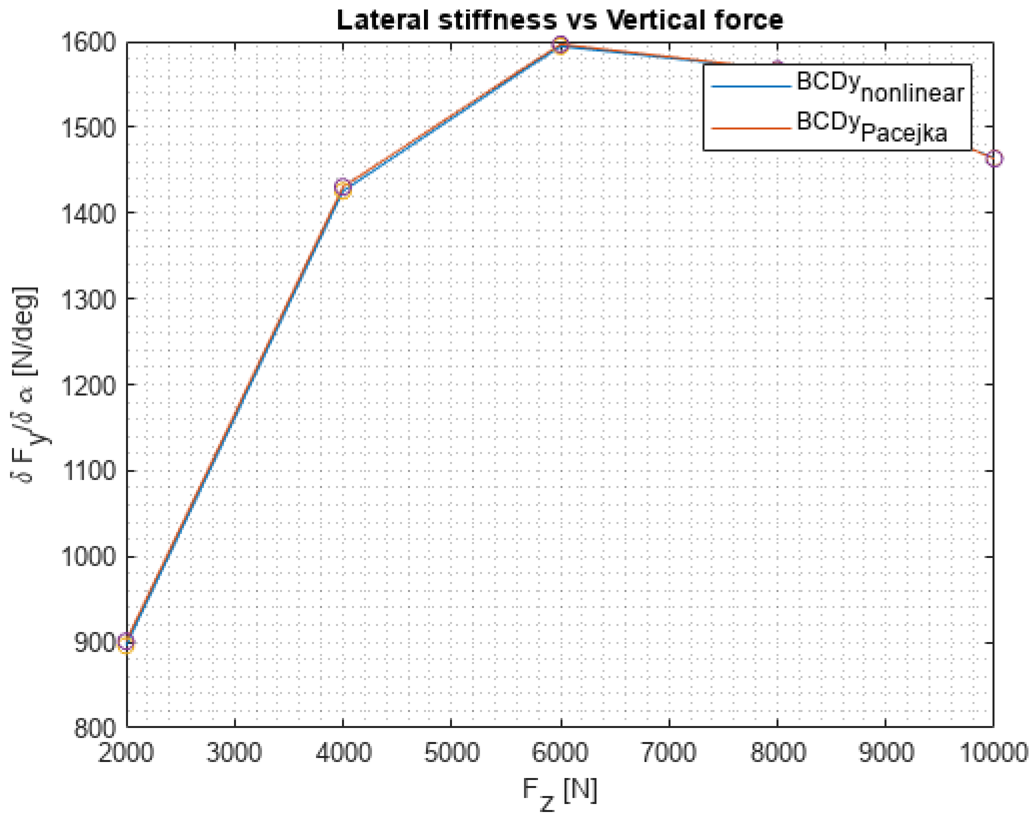
```
Fz = [2:2:10]*10^3;
slopeFx = [];
slopeFy = [];
slopeMz = [];
for index= 1:length(Fz)
    Fx_nonlinear = mu_x(:,index)*Fz(index);
    Fy_nonlinear = mu_y(:,index)*Fz(index);
    Mz_nonlinear = Mz_matrix(:,index);
    % 101 index corresponds to sigma_nonlin=0

    slopeFx = [slopeFx, (Fx_nonlinear(102)-Fx_nonlinear(101))/...
        (sigma_nonlin(102)-sigma_nonlin(101))];
    slopeFy = [slopeFy, (Fy_nonlinear(102)-Fy_nonlinear(101))/...
        (alpha_nonlin(102)-alpha_nonlin(101))];
    slopeMz = [slopeMz, (Mz_nonlinear(102)-Mz_nonlinear(101))/...
        (alpha_nonlin(102)-alpha_nonlin(101))];
end
slopeFx = slopeFx ./ 100; %conversion of sigma in percentage
```

To have a comparison, we plot in three diagrams ($F_x - \sigma, F_y - \alpha, M_z - \alpha$) the values of BCD obtained respectively with Pacejka's and nonlinear models.

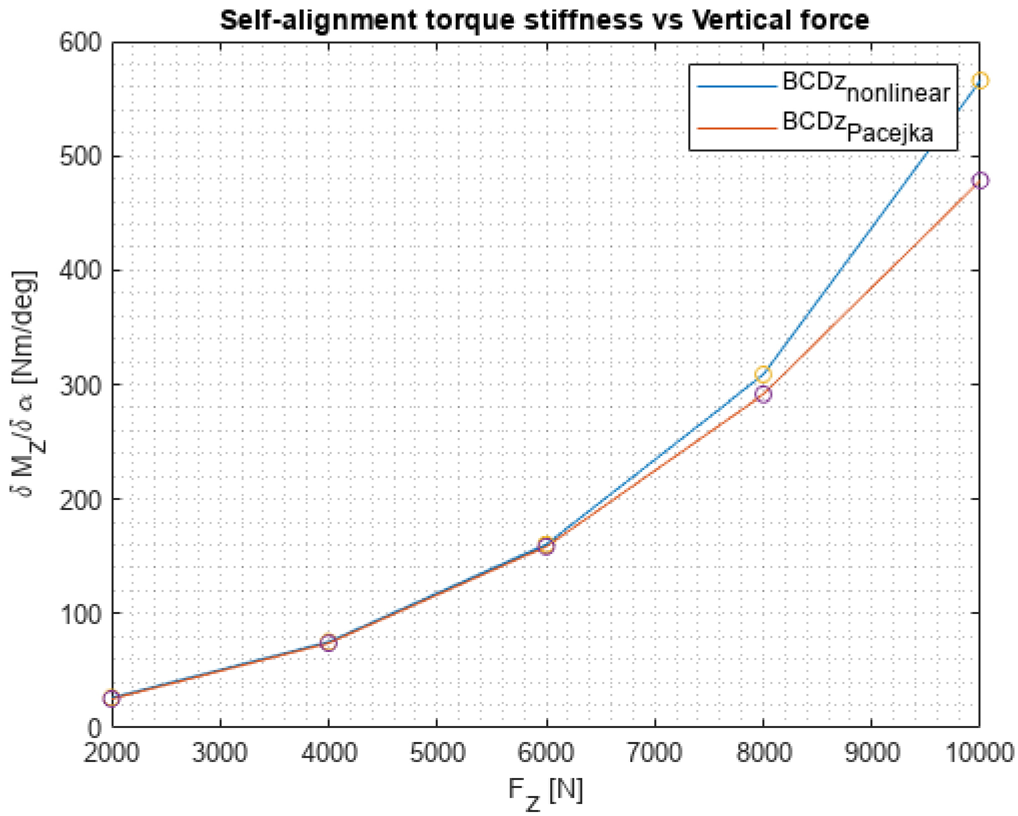
```
% BCDx
figure
plot(Fz,slopeFx); hold on; plot(Fz,BCDx); grid minor
plot(Fz, slopeFx, 'o')
plot(Fz,BCDx,'o');
xlabel('F_z [N]')
ylabel('\delta F_x/\delta \sigma [N]')
legend('BCDx_{nonlinear}','BCDx_{Pacejka}')
title('Longitudinal stiffness vs Vertical force')
```




```
% BCDy
figure
plot(Fz,slopeFy); hold on; plot(Fz,BCDy); grid minor
plot(Fz, slopeFy, 'o');
plot(Fz, BCDy, 'o');
xlabel('F_z [N]')
ylabel('\delta F_y/\delta \alpha [N/deg]')
legend('BCDy_{nonlinear}','BCDy_{Pacejka}')
title('Lateral stiffness vs Vertical force')
```

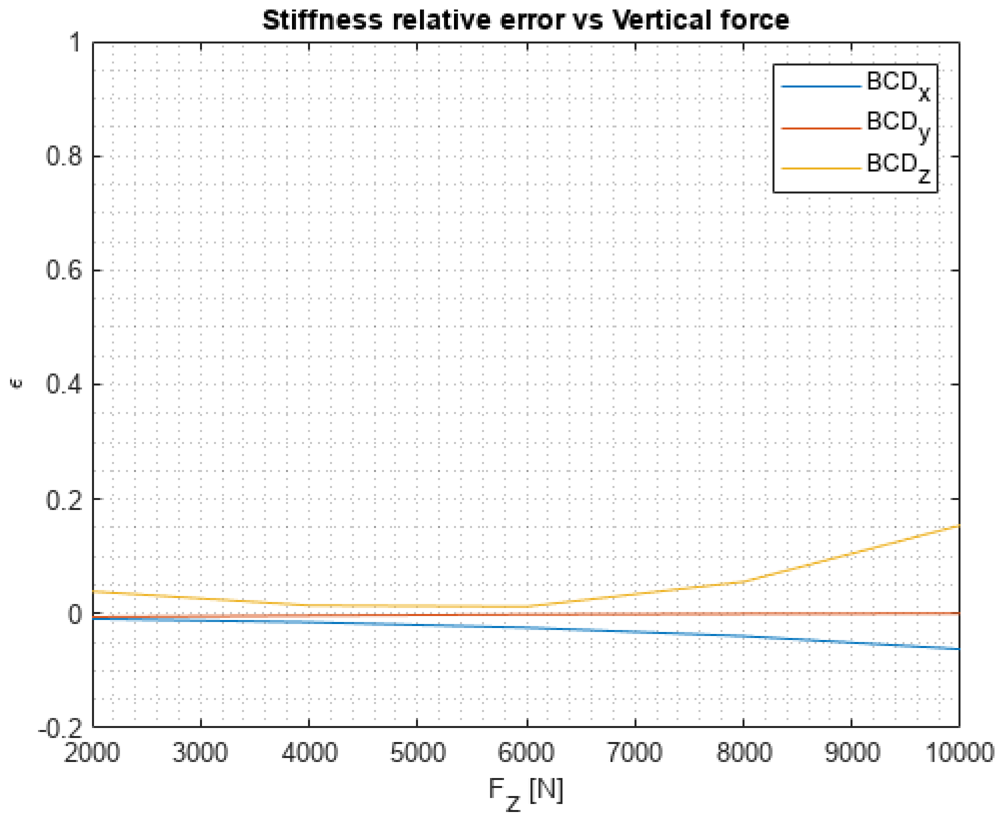



```
% BCDz
figure
plot(Fz,slopeMz); hold on; plot(Fz,BCDz); grid minor
plot(Fz, slopeMz, 'o');
plot(Fz, BCDz, 'o');
xlabel('F_z [N]')
ylabel('\delta M_z/\delta \alpha [Nm/deg]')
legend('BCDz_{nonlinear}','BCDz_{Pacejka}')
title('Self-alignment torque stiffness vs Vertical force')
```



To have a further indicator of the precision of the results obtainded with the Pacejka's model, we introduced a relative error for each stiffness.

```
slope = [slopeFx', slopeFy', slopeMz'];
BCD = [BCDx', BCDy', BCDz'];
% relative error
eps = [];
dimension = size(slope);
for index = 1 : dimension(1,2)
    eps = [eps, (slope(:, index) - BCD(:, index)) ./ slope(:, index)];
end
figure
plot(Fz, eps); grid minor
ylim([-0.2,1]);
xlabel('F_z [N]');
ylabel('\epsilon');
lgd = legend('BCD_x', 'BCD_y', 'BCD_z');
title('Stiffness relative error vs Vertical force')
```



As expected, Pacejka's model matches pretty well the measured behaviour, with a relative error that never exeeds 20%. In addition, we noticed that, for the considered vertical load, the precision of the model is higher for the low/medium levels of force.

3. INTERACTION BETWEEN LONGITUDINAL AND LATERAL TIRE-ROAD FORCES

Pacejkca's Magic Formula provides the behaviour of a tire as the longitudinal and lateral force acts separately. If the tire produces forces along X' and Y' at the same time the behaviour is different because the force along one axis limits the maximum force available along the other axis.

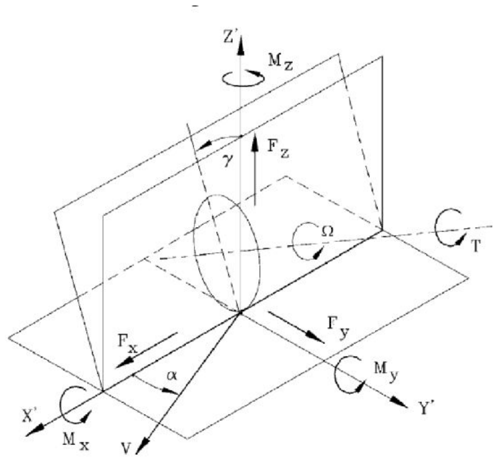


Figure 1 Reference system used to study forces exchanged between tire and ground. Definition of positive direction of forces, moments and side slip angle

Elliptical model offers an approximation to describe this behaviour:

$$\left(\frac{F_y}{F_{y0}}\right)^2 + \left(\frac{F_x}{F_{x0}}\right)^2 = 1$$

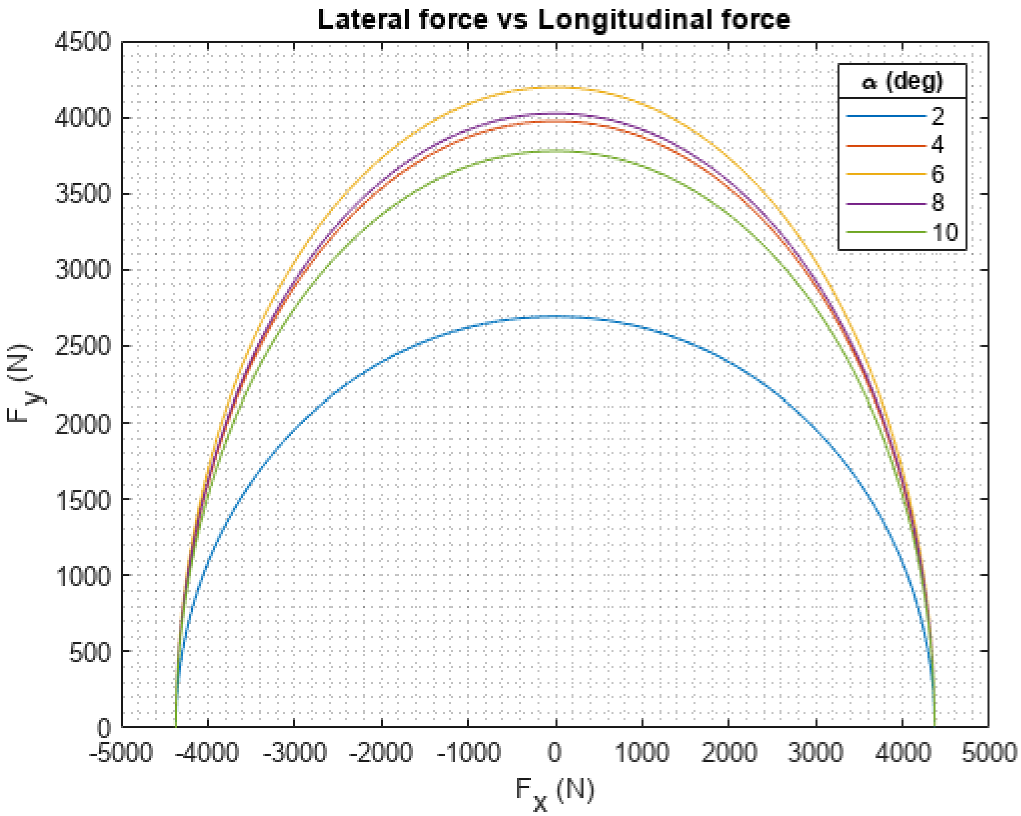
From this we calculate the lateral force F_y as function of the longitudinal force F_x:

$$F_y = F_{y0}\sqrt{1 - \left(\frac{F_x}{F_{x0}}\right)^2}$$

For the calculations we consider $F_z = 4\,\mathrm{kN}$, $\gamma = 0°$, $\alpha = 2°, 4°, 6°, 8°, 10°$.

```
Fy_elliptical = [];
Fz = 4*10^3; % N
mu_y = Fy_matrix(:,2)/Fz;
% Lateral force with elliptical model
Fx0 = Fz * mu_xp(2);
Fx_vector = -Fx0 : 0.1 : Fx0;
Fx_vector = Fx_vector';
for alpha = 111:10:151 % indexes of the considered values of alpha
    Fy_elliptical = [Fy_elliptical, Fz * mu_y(alpha, 1) * ...
        ( 1 - (Fx_vector / (Fz * mu_xp(2))).^2).^(1/2)];
end

figure;
plot(Fx_vector, Fy_elliptical); ; grid minor
title('Lateral force vs Longitudinal force')
xlabel('F_x (N)')
ylabel('F_y (N)')
lgd = legend('2', '4', '6', '8', '10');
```

```
lgd.Title.String = '\alpha (deg)';
```



Then we calculate the lateral friction coefficient μ_y as function of the longitudinal friction coefficient μ_x.

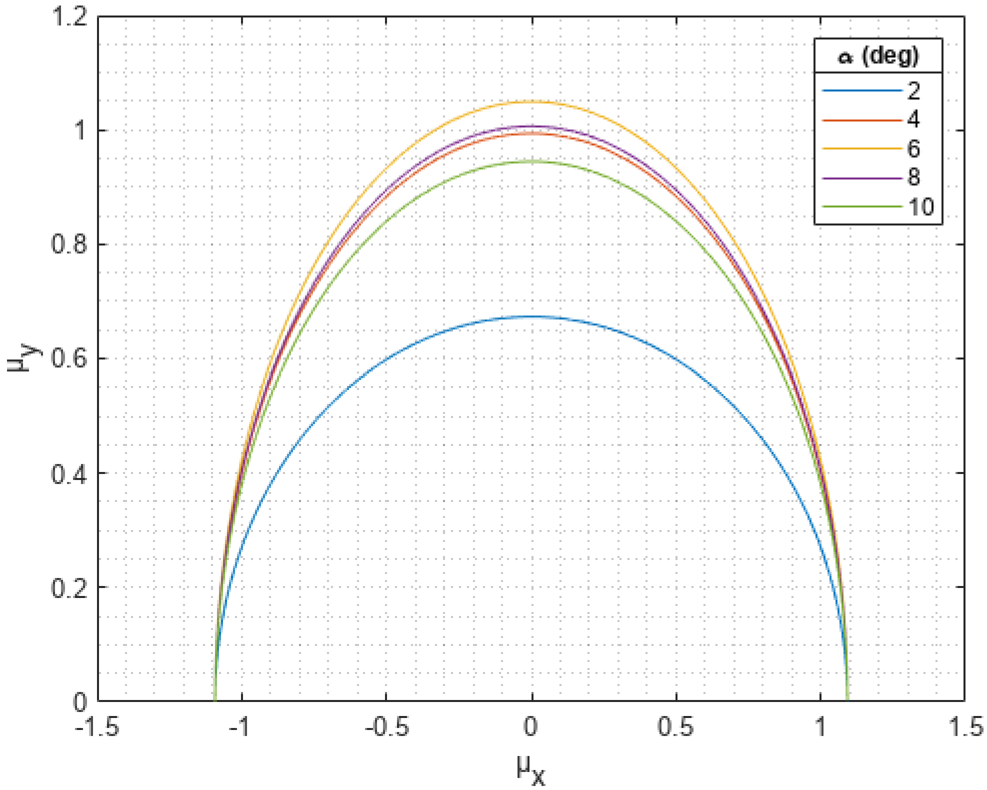
Using the following equation:

$$\mu_y = \mu_{y0}\sqrt{1-\left(\frac{\mu_x}{\mu_p}\right)^2}$$

Where μ_y and μ_{y0} indicate the coefficient along the side direction with and without any longitudinal force.

```
% Lateral friction coefficient with elliptical model
muy_elliptical = [];
Fz = 4*10^3;
mu_x = Fx_matrix(:,2)/Fz;
mux_vector = -mu_xp(2) : 0.1 : mu_xp(2);
mux_vector = mux_vector';
for alpha = 111:10:151 % indexes of the considered values of alpha
   muy_elliptical = [muy_elliptical, mu_yp(2) * ( 1 - (mux_vector / mu_xp(2)).^2).^(1/2)];
end
figure;
plot(Fx_vector/Fz,Fy_elliptical/Fz)
xlabel('μ_x');
ylabel('μ_y'); grid minor
```

```
lgd = legend('2', '4', '6', '8', '10');
lgd.Title.String = '\alpha (deg)';
```



At the end, we evaluate the side slip stiffness C as a function of the longitudinal force F_x, for $F_z = 2, 4, 6, 8, 10\ N$:

$$C = C_0 \sqrt{1 - \left(\frac{F_x}{\mu_{xp} F_z}\right)^2}$$

Where the coefficient C_0 is the cornering stiffness when no longitudinal force is applied.

```
C_elliptical = [];
Fx_plot = [];

Cmatrix = Fy_matrix./alpha_nonlin;
C0 = max(Cmatrix)/1000; % kN/deg
Fz = 2:2:10;
for index = 1:length(Fz)
    Fx0 = mu_xp(index)*Fz(index)*1000;
    Fx_vector = linspace(-Fx0, Fx0, 10^5)';
    Fx_plot = [Fx_plot, Fx_vector];
    C_elliptical = [C_elliptical, (C0(index) * (1 - (Fx_vector/Fx0).^2)).^(1/2)];
end

figure;
```

```
plot(Fx_plot, C_elliptical);grid minor
title('Cornering stiffness vs Longitudinal force')
xlabel('F_x [N]');
ylabel('C_y [kN/deg]')
lgd = legend('2', '4', '6', '8', '10');
lgd.Title.String = 'Fz [kN]';
```