Politecnico di Torino
Department of Mechanical and Aerospace Engineering

MOTOR VEHICLE DESIGN - 04 LNLLI

Automotive Engineering - A.Y. 2022-2023

PROJECT 2.1
Longitudinal Dynamics

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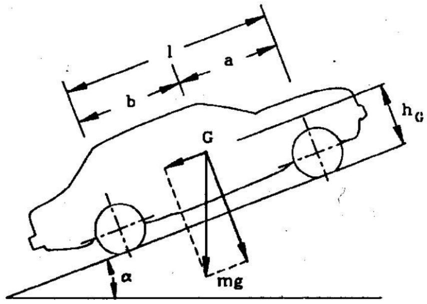
Abstract

This project will analyze, from several points of view, the longitudinal dynamics of a vehicle. Will be first established and visualized the relationships between the available power for motion and the components of the resistive forces against the vehicle traction. This will be performed making some assumptions based on the speed itself of the car and the road grade undergone. From the engine data, an approximation of the power curves is provided by the Artamonov model. Thanks to the results obtained, it has been possible to determine the gradeability of the vehicle and transmission ratios' values.

Power needed for motion




```
close all
clear
clc

V = linspace(0,50,100);
g = 9.81;
rho = 1.3;
f0 = 0.01;
K = 3*10^(-6);
m = 1700;
S = 2.3;
Cx = 0.3;
```

The quantities will be evaluated at 3 different road grades, which affect the result by means of this formula:

$$R = \left(mg\,cos(\alpha) - \frac{1}{2}\rho V^2 S C_z\right)(f_0 + kV^2) + \frac{1}{2}\rho V^2 S C_x + mg\,sin(\alpha),$$

```
for alpha = [atan(0), atan(0.1), atan(0.2)]

    alpha_deg = alpha*180/pi % prints the value of alpha so that is possible to divide them

    A = m*g*(f0*cos(alpha)+sin(alpha));
    B = m*g*K*cos(alpha) + 0.5*rho*S*Cx;

    R = A + B.*V.^2;
```

Thanks to this diagram is possible to observ the **total resistance** and its two composing terms:

```
    figure
    plot(V,R)
    hold on
    plot(V, A*ones(100))
    plot(V, B.*V.^2,"r")
    xlabel("Velocity [m/s]")
```

```
    ylabel("Resistance [N]")
    lgd1 = legend('R', 'B*V^2','A',"Location","northwest");
    grid on
    hold off
```

While the aim of this plot is to evidence the **power needed for motion** and again its two composing terms, that will be denoted with A and B subscripts :

```
    Pn = A.*V + B.*V.^3;

    figure
    loglog(V, Pn)
    hold on
    loglog(V, A.*V)
    loglog(V, B.*V.^3)
    xlabel("log(Velocity) [m/s]")
    ylabel("log(Power) [W]")
    lgd2 = legend ('P_n', 'P_A', 'P_B',"Location","southeast");
    grid on
    hold off
```

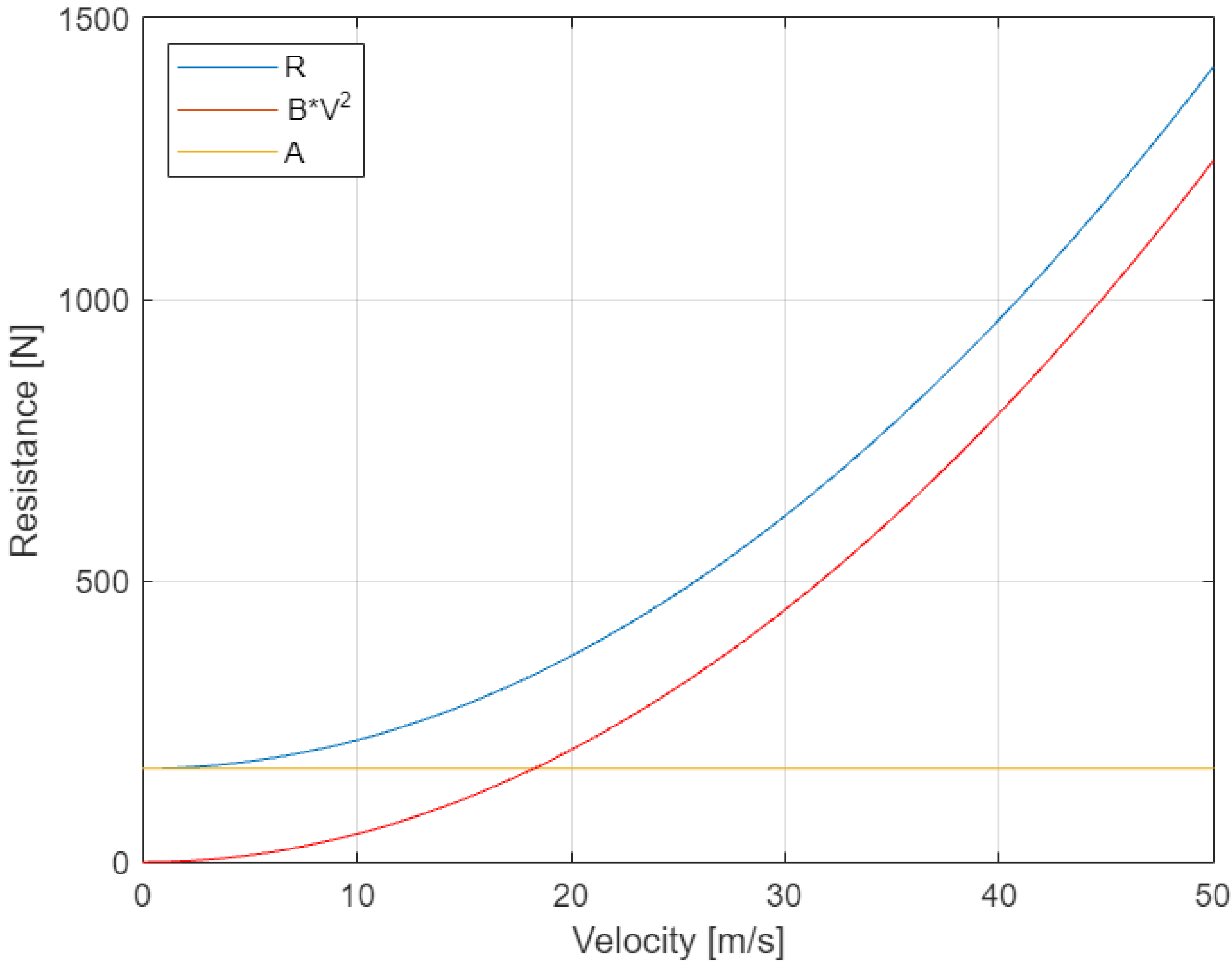
The evaluation of the **characteristic speed** is performed only at *zero slope*:

```
    if alpha == atan(0)
        Vchar = sqrt(A/B); % this speed is evaluated only at alpha = 0°
        Pchar = 2*A*sqrt(A/B);
    end
```

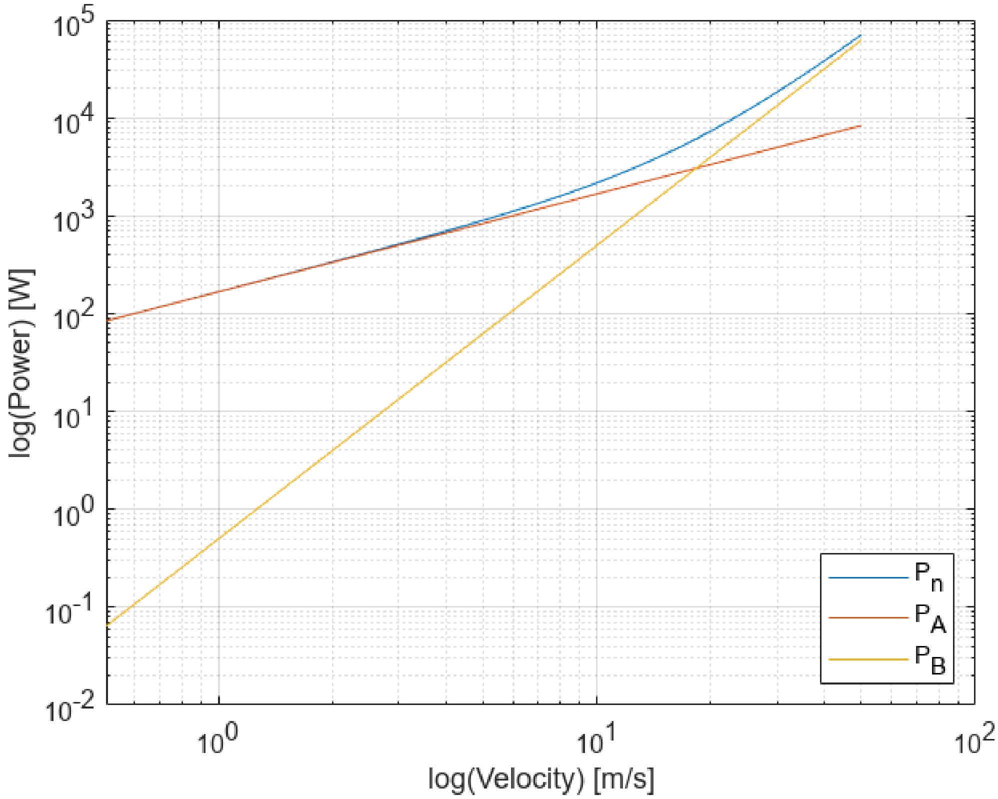
And is now possible to establish the **normalized velocity** and **normalized power**, thanks to the above-mentioned *characteristic speed*:

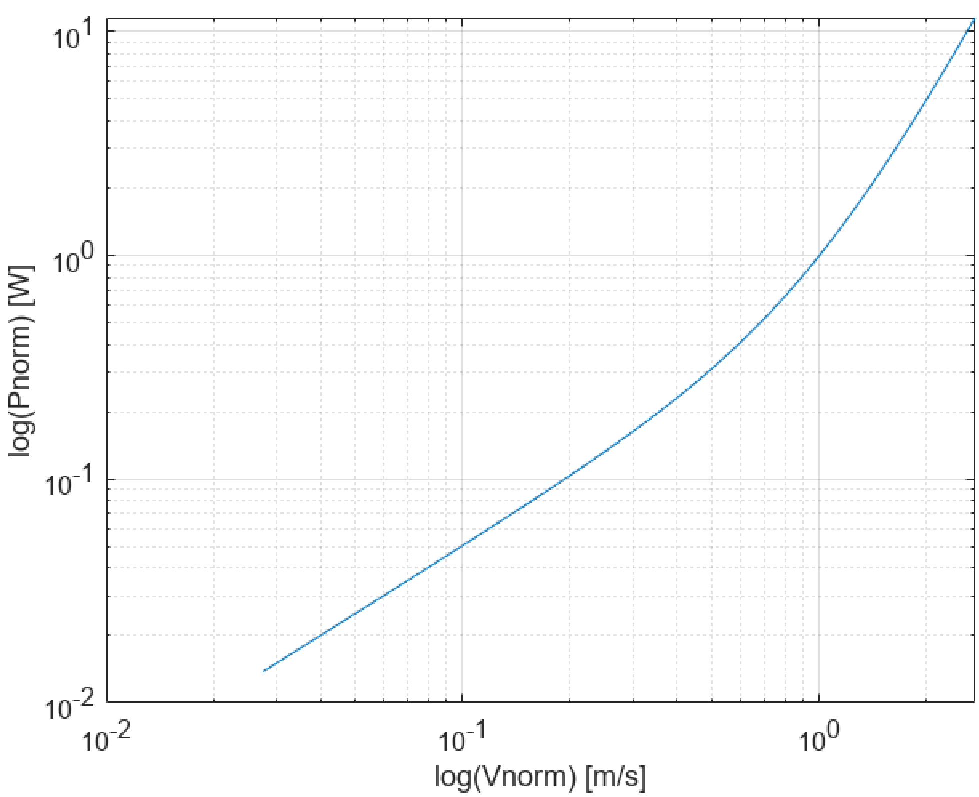
```
    Vnorm = V./Vchar;
    Pnorm = Pn./Pchar;
    figure
    loglog(Vnorm, Pnorm)
    xlabel("log(Vnorm) [m/s]")
    ylabel("log(Pnorm) [W]")
    grid on

end
```

```
alpha_deg = 0
```

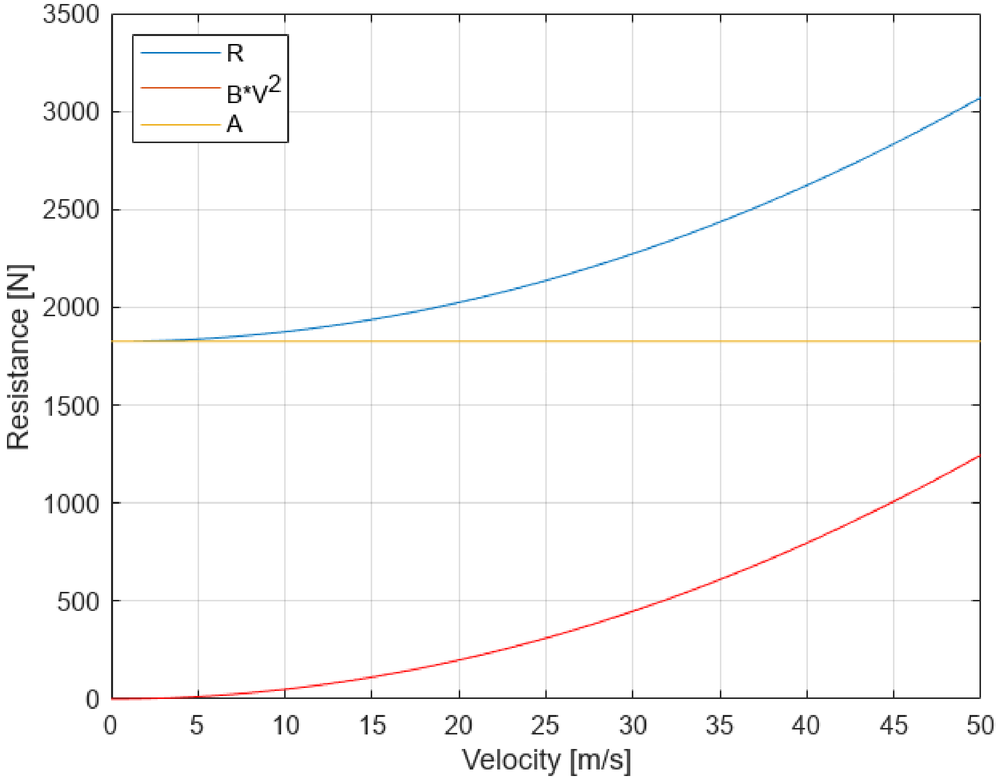


R
B*V^2
A
Resistance [N]
Velocity [m/s]
1500
1000
500
0
0
10
20
30
40
50

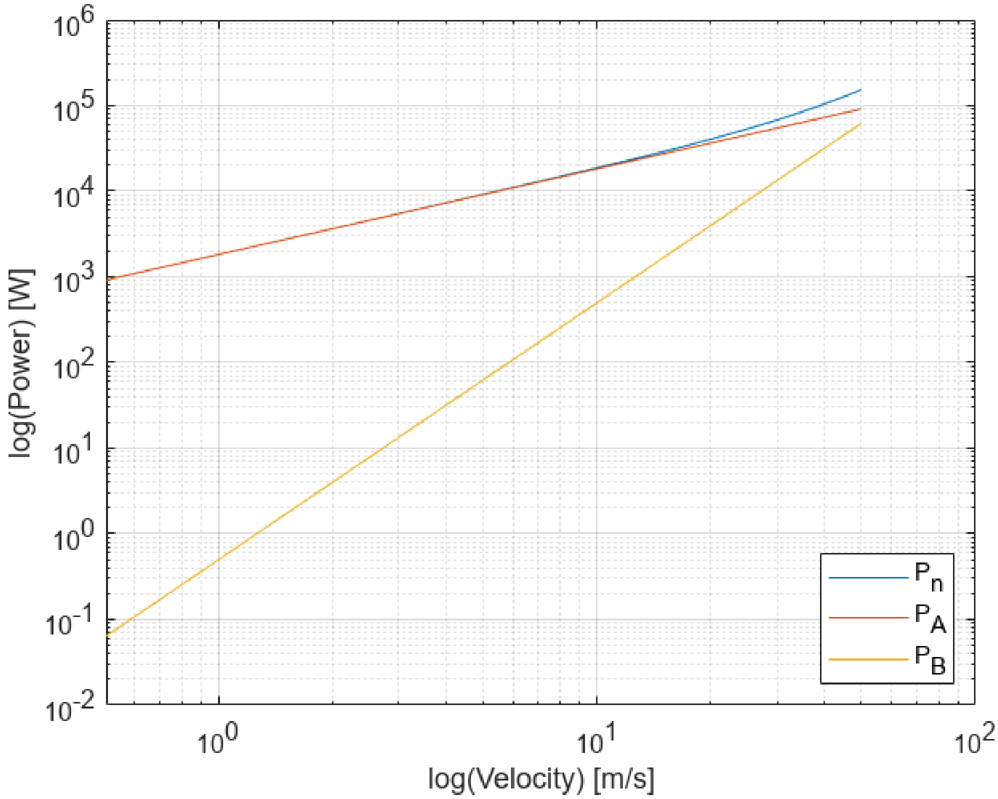




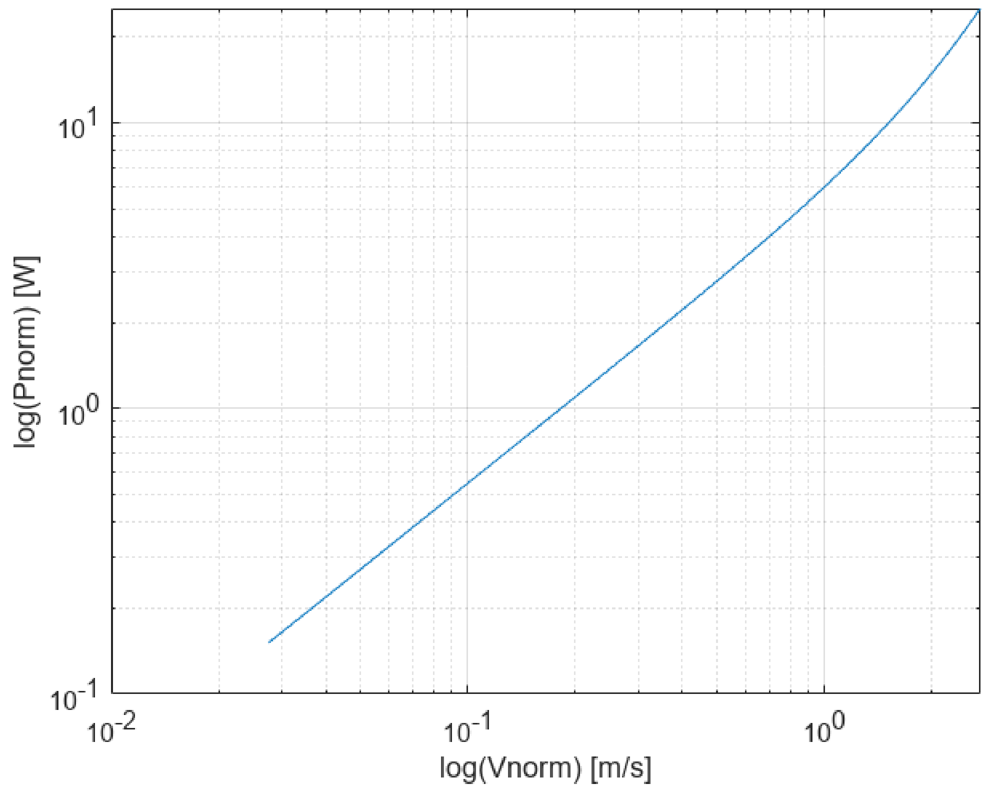

```
alpha_deg = 5.7106
```



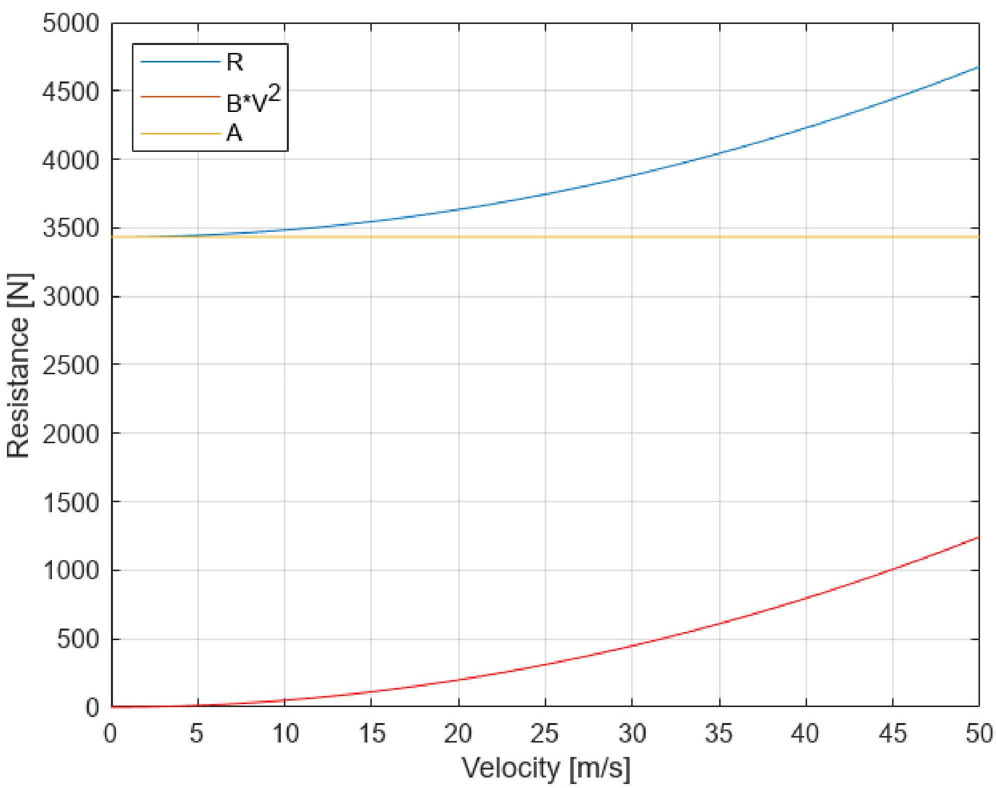
R
B*V2
A
Resistance [N]
Velocity [m/s]

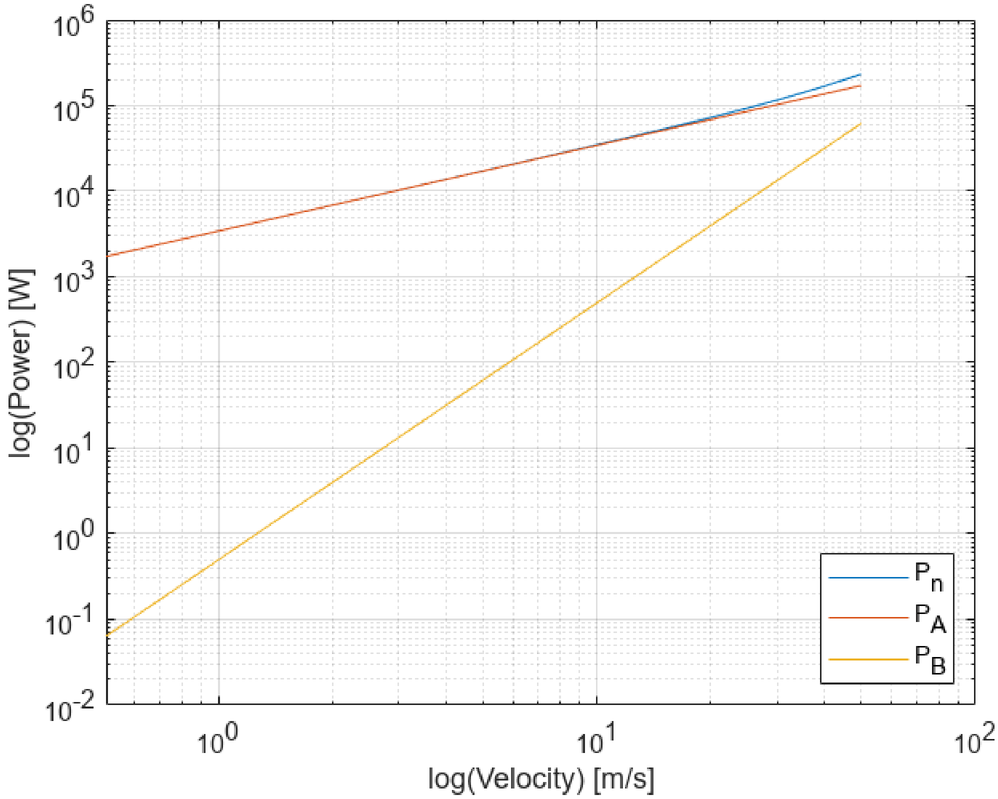


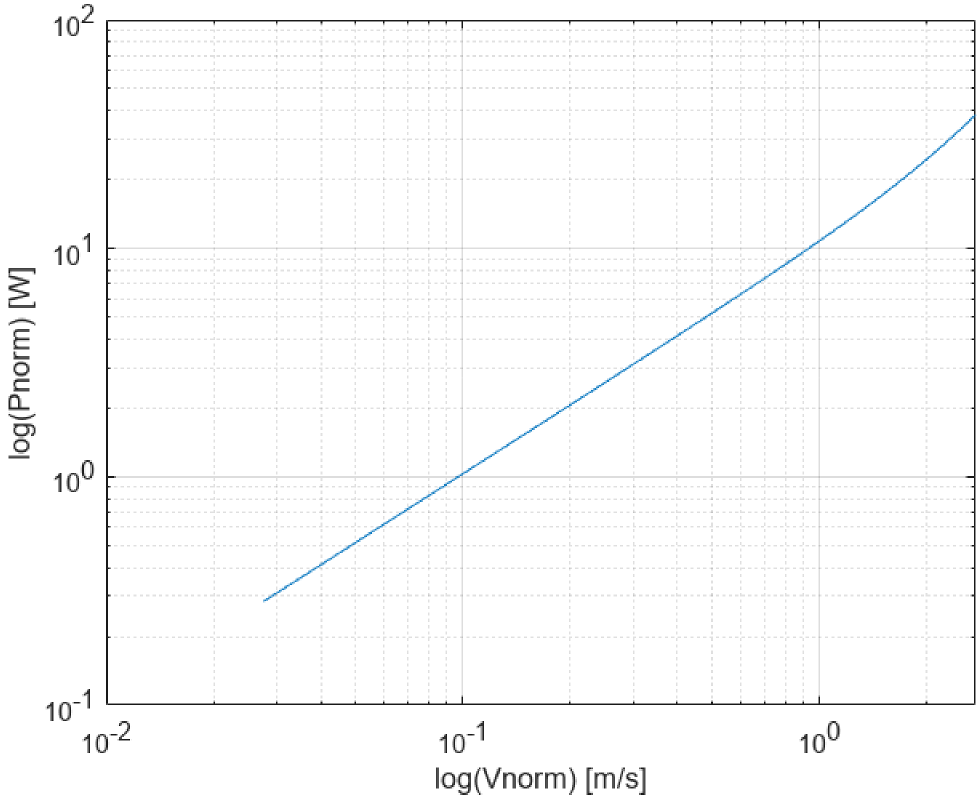
Pn
PA
PB
log(Power) [W]
log(Velocity) [m/s]




```
alpha_deg = 11.3099
```







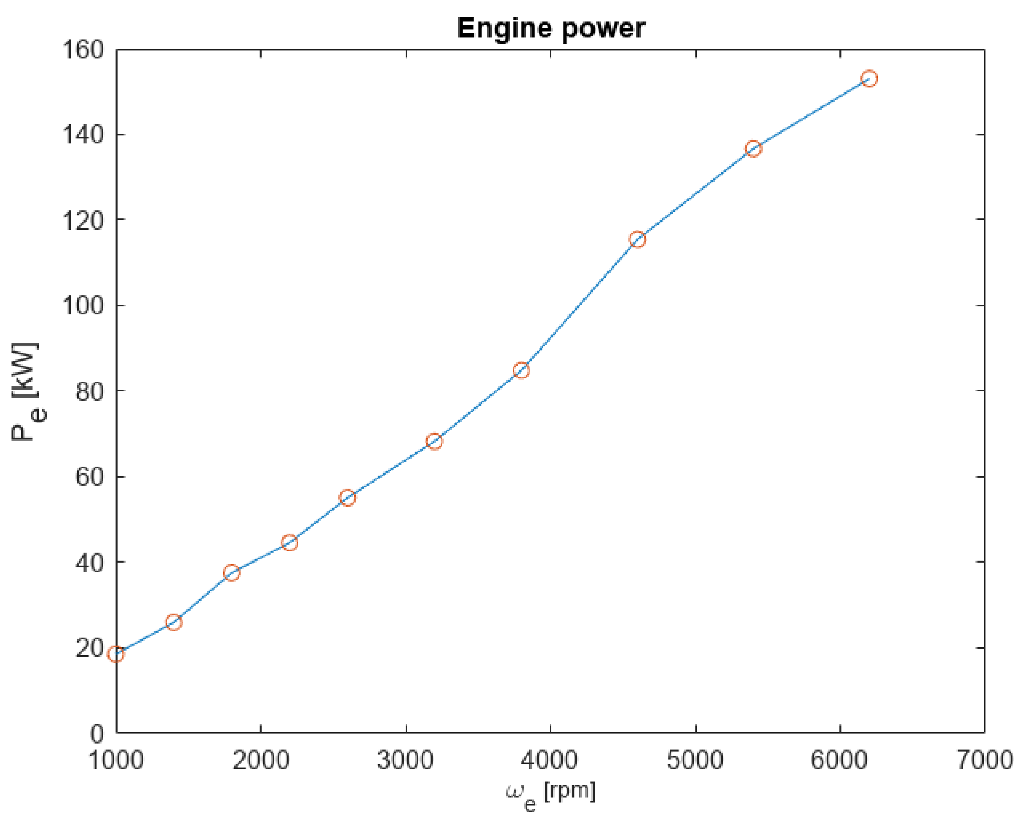
Maximum power available at wheels

```
clear
```

```
clc

Vcil = 2800 * 10^(-6); % m^3
i_strokes = 2;
rpm = readmatrix('rpm_mep_q.xlsx', 'Range', 'B18:K18'); % rad/sec
mep = readmatrix('rpm_mep_q.xlsx', 'Range', 'B19:K19') * 10^5; % Pa
omega = rpm * 2 * pi / 60;

Pe = mep .* Vcil .* omega ./ (2*pi * i_strokes) / 10^3; % kW
```

Thanks to this plot will be possible to visualize the **power** trend as a function of the *rotational speed*, which will be useful furthermore to establish the "ωp,max" term, that is the rotational speed at which the **maximum power** occurs:

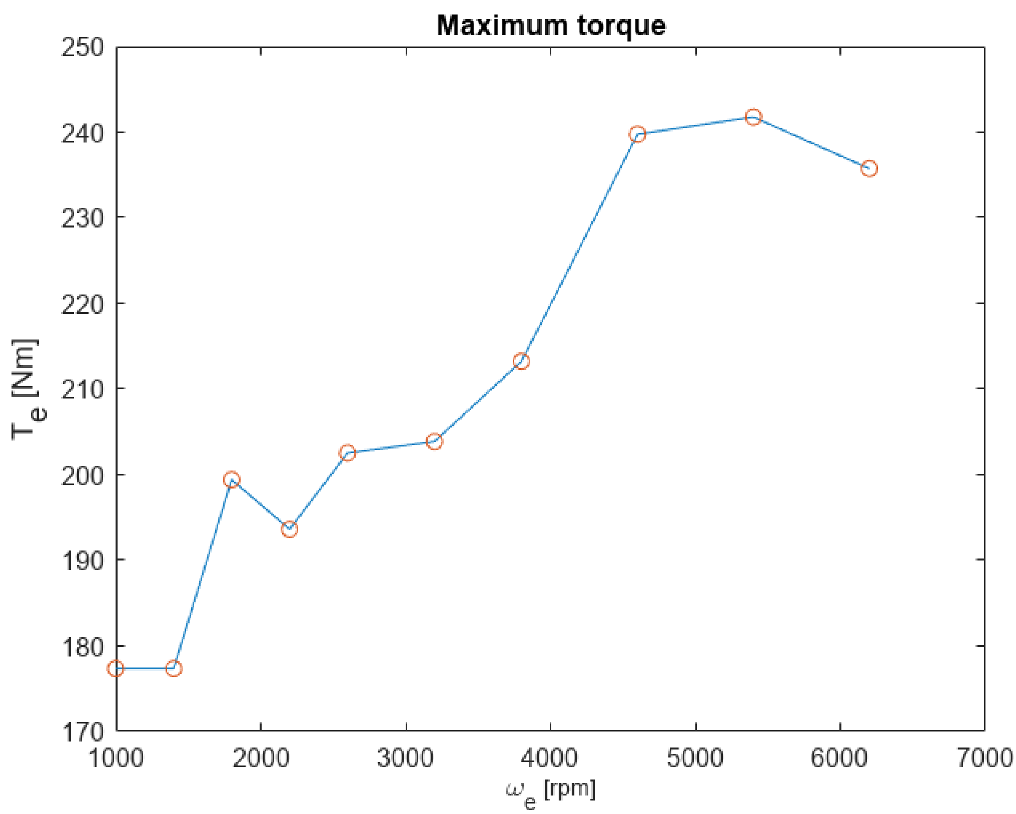
```
figure;
plot(rpm, Pe); hold on; plot(rpm, Pe, 'o');
xlabel('\omega_e [rpm]')
ylabel('P_e [kW]')
title("Engine power")
```



For what concerns instead the **maximum torque**:

```
torque = Pe .*1000 ./ omega; % Nm
figure;
plot(rpm, torque); hold on; plot(rpm, torque, 'o');
xlabel('\omega_e [rpm]')
ylabel('T_e [Nm]')
```

```
title("Maximum torque")
```



We remember that all these quanties have been evaluated under the assumptions of *maximum throttle* conditions.

Here are listed the main quantities evaluated before, such as **maximum power**, the **rotational speed** at which it occurs and the **maximum torque**, which will be defined in order to compute the **power curve**, with the empirical model of *Artamonov*:

```
Pe_max = max(Pe) % kW
```

```
Pe_max = 153.0573
```

```
omega_max = max(omega) % rad/s
```

```
omega_max = 649.2625
```

```
torque_max = max(torque) % Nm
```

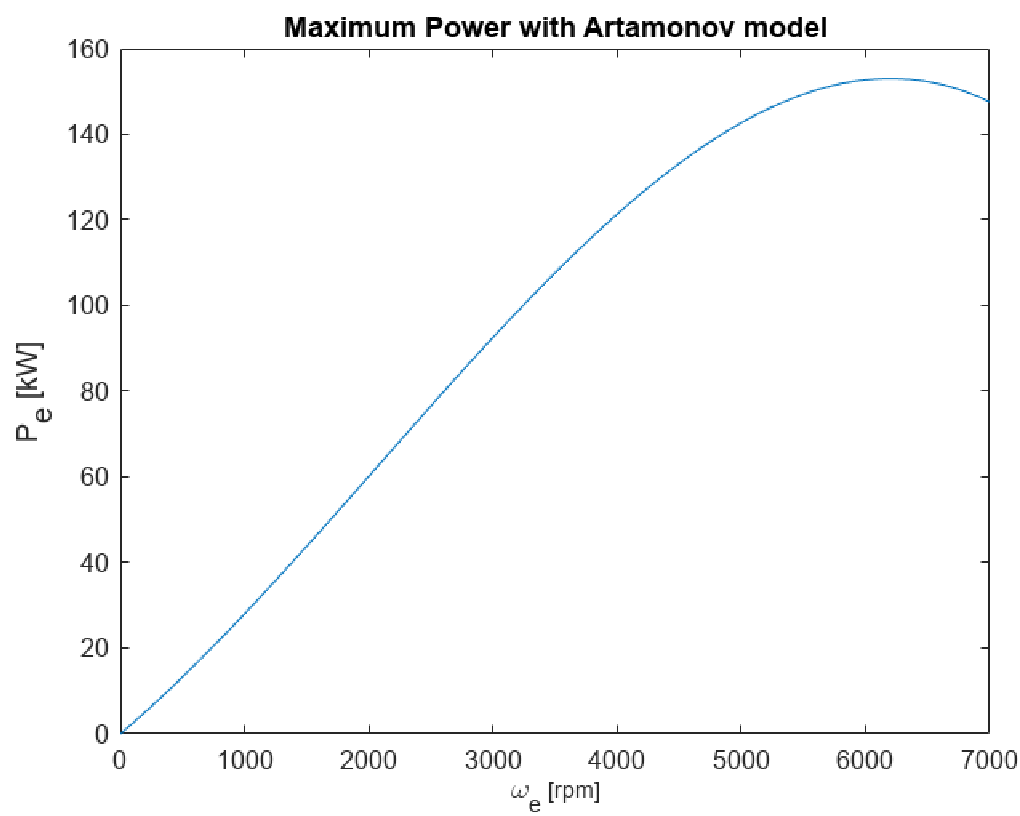
```
torque_max = 241.7564
```

$$P_e = \sum_{i=1}^{3} P_i \omega_e^i$$

where the coefficients P_i are

$P_0 = 0$ $\quad P_1 = P_{max}/\omega_{P,max}$ $\quad P_2 = P_{max}/\omega^2_{P,max}$ $\quad P_3 = - P_{max}/\omega^3_{P,max}$.

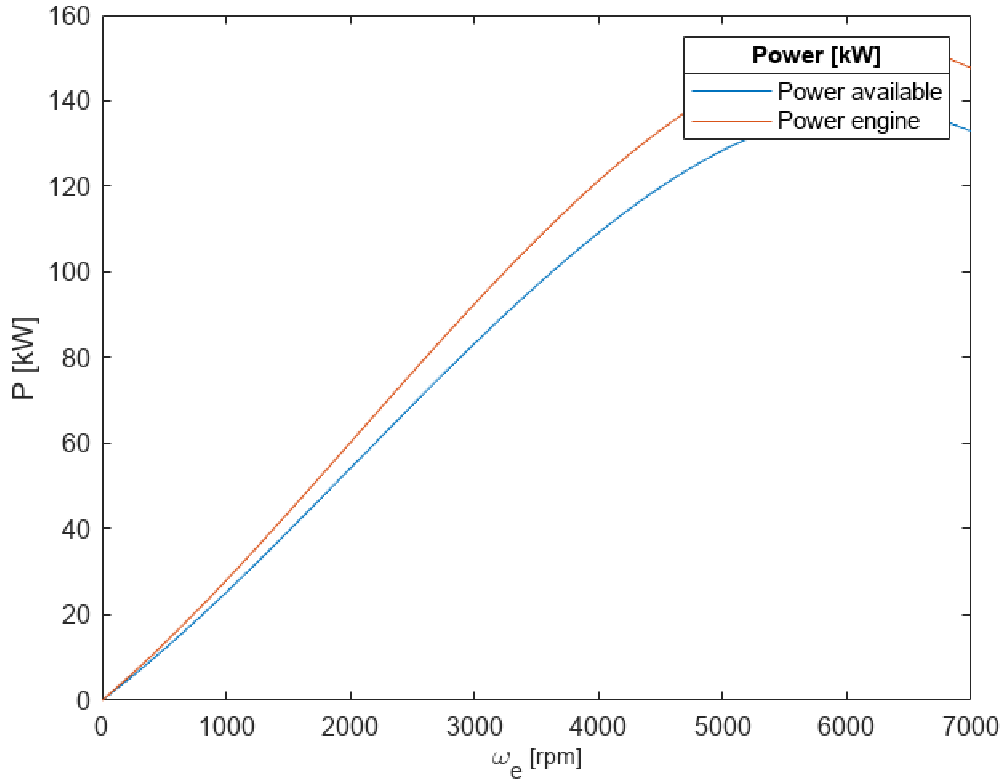
```
rpm = linspace(0,7000); % rpm
omega =  rpm * 2 * pi / 60; % rad/s
Pe = zeros(1,100);

for i = 1:3
    if i == 3
        Pi = -Pe_max/omega_max^i;
    else
        Pi = Pe_max/omega_max^i;
    end
    Pe = (Pe + Pi * omega.^i);
end

figure;
plot(rpm, Pe);
xlabel('\omega_e [rpm]')
ylabel('P_e [kW]')
title("Maximum Power with Artamonov model")
```



Finally it's plotted the curve of the **power available at wheels**:

```
eta_t = 0.9;
Pa = eta_t * Pe;
figure;
plot(rpm, Pa); hold on; plot(rpm, Pe);
xlabel('\omega_e [rpm]')
```

```
ylabel('P [kW]')
lgd = legend('Power available', 'Power engine');
lgd.Title.String = 'Power [kW]';
```



3) Gradeability and initial choice of the transmission ratios

The purpose of the third part of the project 2.1 is to define all the transmission ratios of the vehicle. In order to evaluate them all, is necessary to design before the top and the bottom transmission ratios.

```
close all
clear
clc

m = 1700; % total mass of the vehicle
g = 9.81; % gravity acceleration
f0 = 0.01; % rolling resistance 1
K = 3e-6; % rolling resistance 2
rho = 1.3; % air density
S = 2.3; % cross section area
Cx = 0.3; % aerodinamic drag coefficient
V = 2800*10^(-6); % engine displacement in m^3
i_strokes = 2; % i in mep formula
eta_t = 0.9; % efficiency of the powertrain
tau_f = 0.2679; % differential transmission ratio
R = ((16*25.4/2) + 0.55*205)*10^(-3); % non loaded radius of the wheel in m
R_L = 0.98 * R; % loaded radius of the wheel

n_engine = [1000; 1400; 1800; 2200; 2600; 3200; 3800; 4600; 5400; 6200];
% engine speed in rpm from the engine map file
omega_engine = n_engine*2*pi/60; % engine speed in rad/s
mep_fullthrottle = [7.96; 7.96; 8.95; 8.69; 9.09; 9.15; 9.57; 10.76; 10.85; 10.58] * 10^5;
% max value of mep (in Pa) for each engine speed from the engine map file
P_e_fullthrottle = (V*mep_fullthrottle.*omega_engine)/(2*pi*i_strokes);
% max power for each engine speed
p_a_fullthrottle = eta_t*P_e_fullthrottle; % maximum values of available power
% for each engine speed
P_emax = max(P_e_fullthrottle); % max power of the engine
P_amax = eta_t * P_emax; % max power available at the wheels
```

Once the maximum power is evaluated it is possible to proceed applying the Cardano's formula reported below.

This method allows us to compute the top gear ratio from the knowledge of the parameters of the vehicle and the maximum power provided by the internal combustion engine.

$$V_{max} = A^*\left(\sqrt[3]{B^*+1} - \sqrt[3]{B^*-1}\right)$$

$$A^* = \sqrt[3]{\frac{P_{a,max}}{2m\,g\,K + \rho\,S\,C_x}} = \sqrt[3]{\frac{P_{a,max}}{2B}}$$

$$B^* = \sqrt{1 + \frac{8\,m^3 g^3 f_0^3}{27\,P_{a,max}^2 (2m\,g\,K + \rho\,S\,C_x)}} = \sqrt{1 + \frac{4\,A^3}{27\,P_{a,max}^2\,B}}$$

$$\tau_{g,top} = \frac{V_{max}}{\tau_f R_L \omega_{P,max}}$$

```
A_star = (P_amax/(2*m*g*K + rho*S*Cx))^(1/3);
% coefficient A* (Cardano's formula)
B_star = (1+(8*m^3*g^3*f0^3)/(27*P_amax^2*(2*m*g*K + rho*S*Cx)))^(1/2);
% coefficient B* (Cardano's formula)
V_max = A_star*((B_star + 1)^(1/3) - (B_star - 1)^(1/3)); % Cardano's formula, V_max in m/s

for index = 1:length(omega_engine)% the cycle matches the maximum power of the
    % engine with its correspondent engine speed
    if P_e_fullthrottle(index) == P_emax
        omega_pmax = omega_engine(index);
    end
end

tau_gtop = V_max/(tau_f * R_L * omega_pmax); % top transmission ratio

M_eminspeed = P_e_fullthrottle(1)/omega_engine(1); % maximum torque at minimum engine speed
i_max = 0.33; % maximum road grade
alpha_max = atan(i_max);
margin = 0.7; % margin of reduction of the bottom gear ratio
```

The calculation of the bottom gear ratio is carried on by the represented formula. From theory it is possible to say that the bottom gear ratio is defined by the maximum torque that the engine can provide at minimum speed.

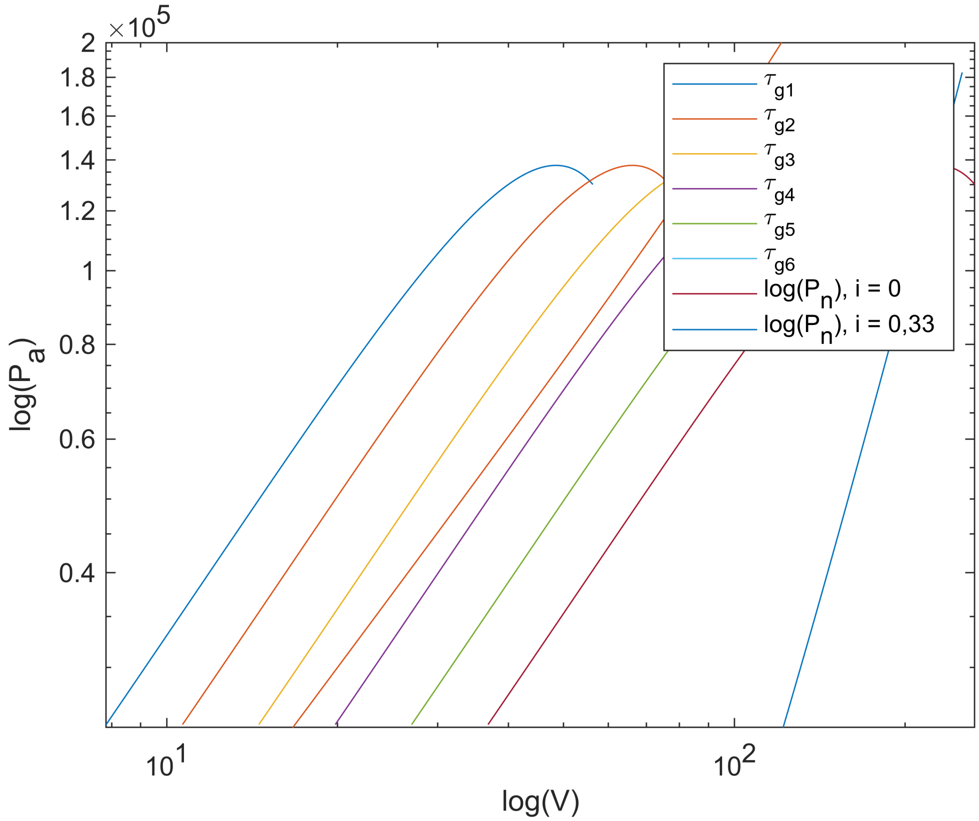
$$\tau_{g,bottom} = \frac{M_e \eta_t}{\tau_f\,R_L\;m\,g\,(f_0 \cos(\alpha_{max}) + \sin(\alpha_{max}))}$$

```
tau_gbottom = margin*(M_eminspeed*eta_t)/(tau_f*R_L*m*g*(f0*cos(alpha_max)+sin(alpha_max)));
% bottom transmission ratio
V_min = omega_engine(1)*tau_gbottom*tau_f*R_L * 3.6;
% speed of the vehicle in km/h at min rotational speed of the engine,
% verified that V_min < 8 km/h
N = 6; % number of gear ratios
tau_g = [tau_gbottom];
```

```
for index = 1:(N-1)        % find all the transmission ratios
    tau_gi = tau_g(index)*(tau_gtop/tau_gbottom)^(1/(N-1));
    tau_g = [tau_g, tau_gi];
end
tau_g = [tau_g, tau_gi]
```

```
tau_g = 1×7
    0.2502    0.3410    0.4649    0.6337    0.8639    1.1776    1.1776
```

```
% power available approximated with Artamonov
P_0 = 0;
P_1 = P_emax/omega_pmax;
P_2 = P_emax/omega_pmax^2;
P_3 = -P_emax/omega_pmax^3;

n_engine_Artamonov = linspace(1000, 7200, 100)'; % extend the range of speed of the engine
omega_engine_Artamonov = n_engine_Artamonov * (2*pi)/60;

V_vehicle_Artamonov = []; % speed of the vehicle in km/h
for index = 1:length(tau_g)
    V_vehicle_Artamonov_i = omega_engine_Artamonov*tau_g(index)*tau_f*R_L*3.6;
    V_vehicle_Artamonov = [V_vehicle_Artamonov, V_vehicle_Artamonov_i];
end

P_e_Artamonov = P_1*omega_engine_Artamonov + P_2*omega_engine_Artamonov.^2 +...
P_3*omega_engine_Artamonov.^3;
P_a_Artamonov = P_e_Artamonov*eta_t;

loglog(V_vehicle_Artamonov, P_a_Artamonov);
xlabel("log(V)")
ylabel("log(P_a)")

alpha_vec = [0, atan(i_max)]; % slope 0 and max slope
V_vec = linspace(0,70, 100);
V_vec_plot = 3.6*V_vec; % speeds in km/h
P_n = [];
for index = 1:2
    A = m*g*(f0*cos(alpha_vec(index))+sin(alpha_vec(index)));
    B = m*g*K*cos(alpha_vec(index))+0.5*rho*S*Cx;
    P_ni= A*V_vec + B*V_vec.^3;
    P_n = [P_n; P_ni];
end

hold on
loglog(V_vec_plot,P_n); ylim([25000,200000]);
legend("\tau_{g1}","\tau_{g2}","\tau_{g3}","\tau_{g4}", ...
```




```
    "\tau_{g5}","\tau_{g6}","log(P_n), i = 0","log(P_n), i = 0,33");
save("var_2_1_3.mat")
```

The analysis ends up with the rapresentation of all the transmission ratios of the vehicle.

It is possible to see the available power provided by each gear with respect to the speed of the vehicle.

In addition it is possible to observ the different values of power needed depending on two different values of road grade

PROJECT 2.2
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Abstract:

The aim of this project is to start understanding what is the maximum power which the tires are capable to transfer, and then computing the acceleration performances of the vehicle under analysis. Finally the computation of the vehicle's fuel consumpution is carried on, based on the NEDC (New European Driving Cylce).

1) Maximum power transferred by the tires:

```
tire_spec = [205, 0.55, 16]; % [W, H/W, D (inches)]
speed_kmh = linspace(0,500); % km/h
speed_ms = speed_kmh / 3.6; % m/s
```

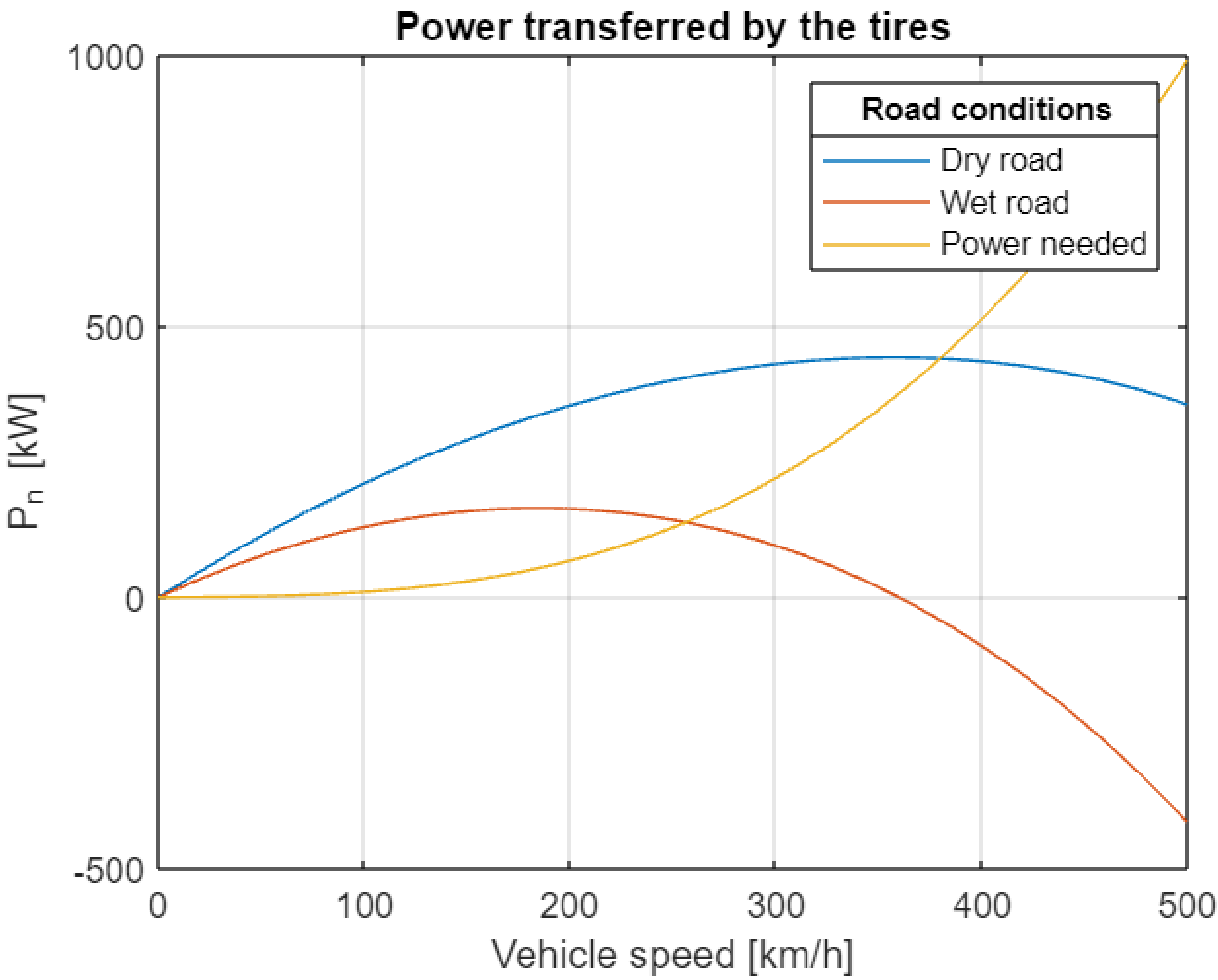
Parameters of the car:

```
m = 1700; % whwight of the car [kg]
l = 2.7; % wheel base [m]
a = 1.3; % distance front axle [m]
b = l-a; % distance rear axle [m]
hg = 0.45; % height of the center of gravity
f0 = 0.01;
K = 3*10^-6; % [s^2/m^2]
g = 9.81; % gravity acceleration [m/s^2]
```

```
Cx = 0.3;
rho = 1.3; % air density [kg/m^3]
S = 2.3; % Section surface [m^2]
Je = 0.35; % Moment of inertia of the engine
Jw = 3; % Momwnt of inertia of wheels and power train
Jt = 0; %supposed

H1 = rho * S * Cx * hg / (2*m*g);
H2 = -H1;

Rl = 0.92 * (tire_spec(3)*25.4/2 + tire_spec(1)*tire_spec(2))*10^-3;
dx = Rl * (f0 + K*speed_ms.^2);

road_coefficient = [1.1 6*10^-3 ; 0.8  8*10^-3];
mu_xp = [];
for i = 1:2
    mu_xp = [mu_xp; road_coefficient(i, 1) - road_coefficient(i,2) * speed_ms]; % [dry; wet]
end

Fz1 = m*g * (b-dx-H1*speed_ms.^2) / l;
Fz2 = m*g * (a+dx-H2*speed_ms.^2) / l;
```

The power transmitted from the tires is analysed in both the cases of dry and wet road:

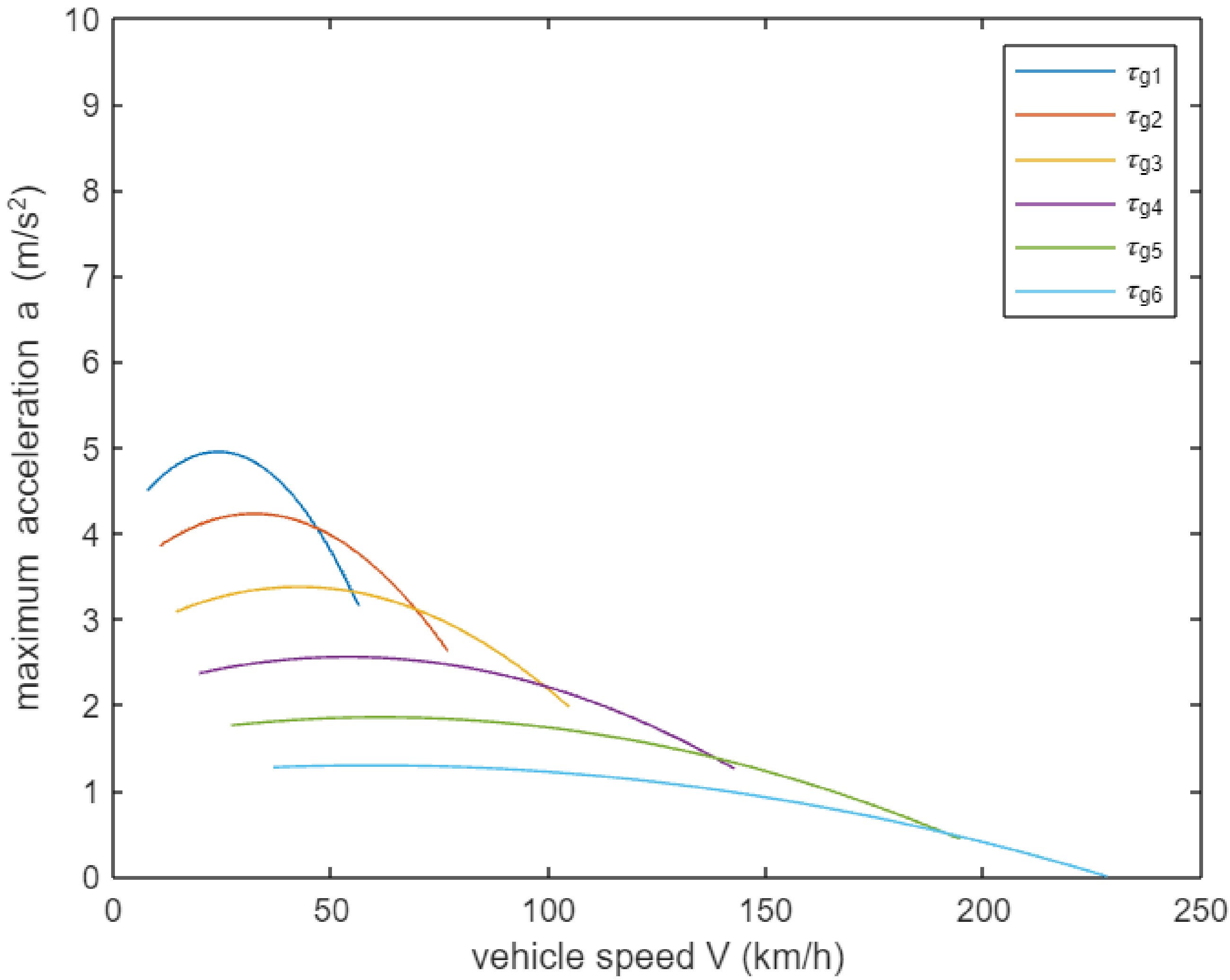
```
Pmax_tires = [];
for i = 1:2
    Pmax_tires = [Pmax_tires; speed_ms .* mu_xp(i, :) .* Fz2]; % Rear wheel drive
    % supposed, so we used rear vertical force;
end
figure;
plot (speed_kmh,Pmax_tires/1000);
xlabel('v [km/h]')
ylabel('P_{max} tires [kW]')
grid on
hold on

Pn = 0.5*S*Cx*speed_ms.^3+Fz1.*(f0+K*speed_ms.^2).*speed_ms; %supposed rear wheel drive [W],
% so we used front vertical force;
plot (speed_kmh,Pn/1000);
xlabel('Vehicle speed [km/h]')
ylabel('P_{n} [kW]')
grid on
lgd = legend('Dry road', 'Wet road', 'Power needed');
lgd.Title.String = 'Road conditions';
title('Power transferred by the tires')
```



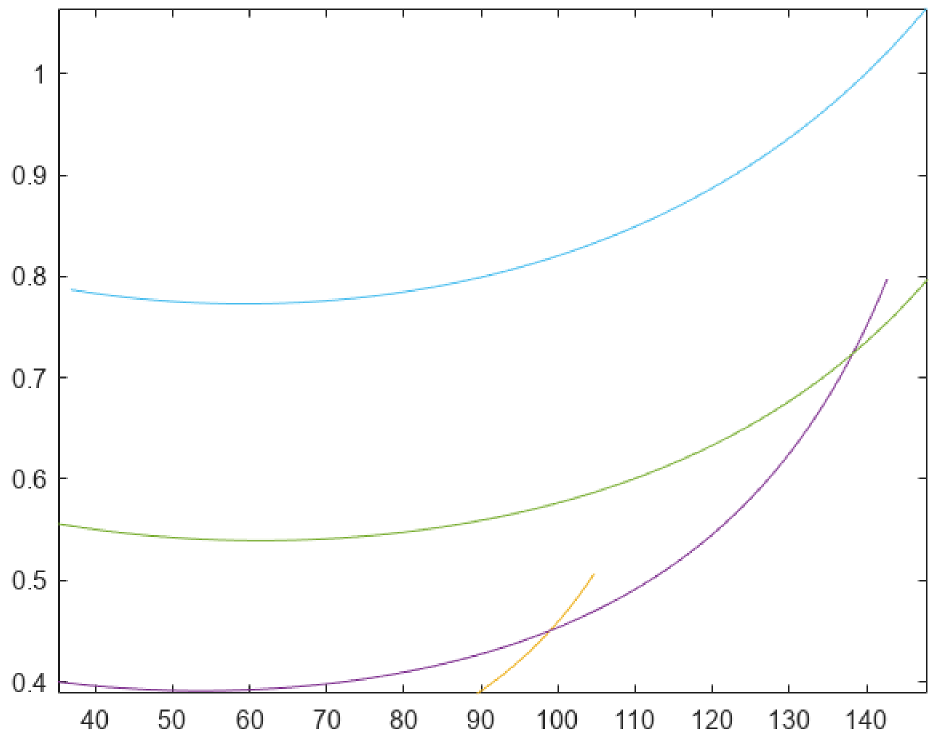
By reading the graph it's possible to notice the different capability of transferring power by the tires in the two different conditions: in dry road conditions the power transferred is always major than that one for wet conditions.

2) Acceleration performance:

```
load("var_2_1_3.mat")

J_w = 3; % moment of inertia of wheels
J_e = 0.35; % moment of inertia of the engine

mass_eq = [];       % equivalent mass for each value of tau_g
for index = 1:length(tau_g)
    mass_eqi= m + J_w/(R_L)^2 + J_e/(R_L^2*tau_f^2*tau_g(index)^2);
    mass_eq = [mass_eq, mass_eqi];
end

Power_needed = [];
for index = 1:length(tau_g)
    V_vehicle_Artamonov_gi = V_vehicle_Artamonov(:, index)/3.6;
    A = m*g*f0;
    B = m*g*K+0.5*rho*S*Cx;
    Power_needed_i = A*V_vehicle_Artamonov_gi + B*V_vehicle_Artamonov_gi.^3;
    Power_needed = [Power_needed, Power_needed_i];
end

P_a_Artamonov_matrix = ones(100,6).*P_a_Artamonov;
mass_eq_matrix = ones(100,6).*mass_eq;
a_max = (P_a_Artamonov_matrix - Power_needed)./((V_vehicle_Artamonov/3.6).* mass_eq_matrix);
figure
plot(V_vehicle_Artamonov, a_max); ylim([0,10]);
xlabel('vehicle speed V (km/h)')
ylabel('maximum acceleration a (m/s^2)')
legend('\tau_{g1}', '\tau_{g2}', '\tau_{g3}', '\tau_{g4}', '\tau_{g5}', '\tau_{g6}')
```




```
a_max_reciprocal = a_max.^-1;
figure;
plot(V_vehicle_Artamonov, a_max_reciprocal);xlim([0 150]);
```




```
syms v
gear_shift_speeds = [];
inferior = V_min/3.6;
area = 0;
for index = 1:6
    sup = V_max / 3.6;
    if index ~= 6
        v_cap = unique([V_vehicle_Artamonov(:,index); V_vehicle_Artamonov(:,index+1)]);
        a_cap = spline(V_vehicle_Artamonov(:,index), a_max(:,index), v_cap);
        a_next_cap = spline(V_vehicle_Artamonov(:,index+1), a_max(:,index+1), v_cap);
        [dist_i, ind_i] = min(abs(a_next_cap-a_cap));
        sup = min(v_cap(ind_i), 100)/3.6;
    end

    omega_e = v/(R_L*tau_f*tau_g(index));
    a_max_reciprocal_i = (v*mass_eq(index))/((P_1*omega_e + P_2*(omega_e^2) ...
        + P_3*(omega_e^3))*eta_t - (A*v + B*(v^3)));
    area_i = double(int(a_max_reciprocal_i, v, inferior, sup ));
    area = area + area_i;
    if abs(sup-100/3.6) < 0.01
        break
    end
    inferior = sup;
    gear_shift_speeds = [gear_shift_speeds, v_cap(ind_i)];
end
t_theoretical = area
```

```
t_theoretical = 7.2082
```

```
shift_time = 1; % Time assumed for the gear shift
t_real = t_theoretical+shift_time*length(gear_shift_speeds);
```